FNU FIJI NATIONAL UNIVERSITY

COLLEGE OF ENGINEERING, SCIENCE AND TECHNOLOGY

School of Electrical & Electronics Engineering

Bachelor of Engineering (Honours) Electrical/Electronics Engineering

EEB722 – Control Systems

FINAL EXAMINATION

Semester 2, 2019

Date: As per Exam Time Table

Time: As per Exam Time Table

Venue: As per Exam Timetable

Duration of Exam: 3 Hours

Total Number of pages: 6

Instructions to Students

1. You are allowed an extra ten (10) minutes of reading time during which you are NOT allowed to write.
2. Attempt all questions.
3. Write your answers in the answer booklet provided.
4. Write your Student ID number on each page used.
5. Begin each Section on a fresh page and use both sides of the answer sheet.
6. You may use calculators provided they are non-programmable.
7. Clearly number the questions in your answer paper in their correct sequence and write legibly. Show all working.
8. Attach any extra sheets used to your answer booklet securely with the string provided.

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Question 1 (15 marks)

a) A conveyor belt speed control system is to be designed for a packaging plant. The conveyor is to be driven by an electric motor. An actuator varies the power to the motor that drives the conveyor. Draw a computer controlled and a non-computer controlled functional closed-loop block diagram of the system and identify the sensor and actuator that could be used. ***(4 marks)***

b) Reduce the block diagram shown in Figure 1 to its canonical form. ***(3 marks)***

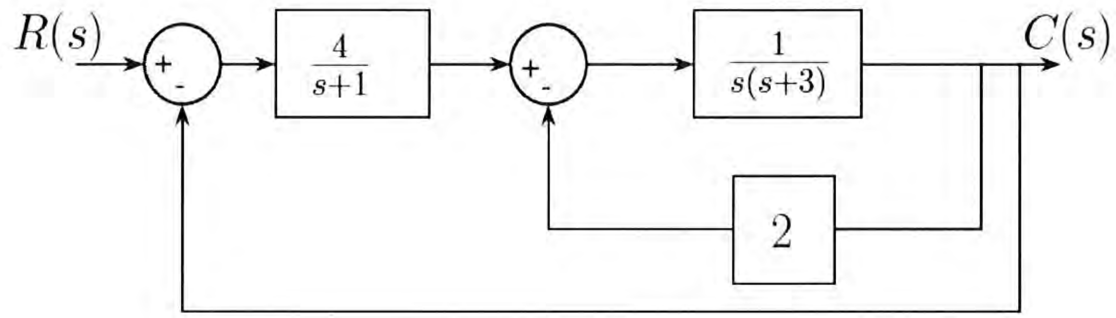

Figure 1

c) For the translational mechanical system shown in Figure 2 below, determine the transfer function $G(s) = \frac{X_2(s)}{F(s)}$. Assume zero initial conditions. ***(8 marks)***

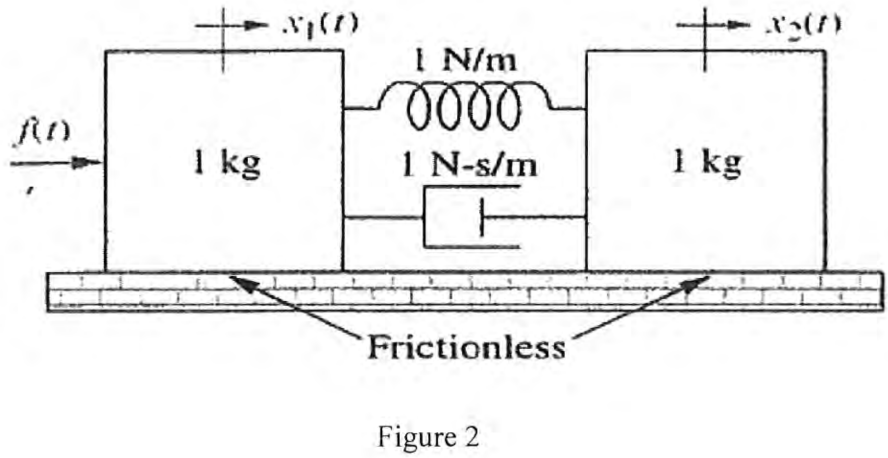

Figure 2

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Question 2 (15 marks)

a) Find the state-space representation for the electrical network system shown in Figure 3, if output is the current through the resistor. ***(7 marks)***

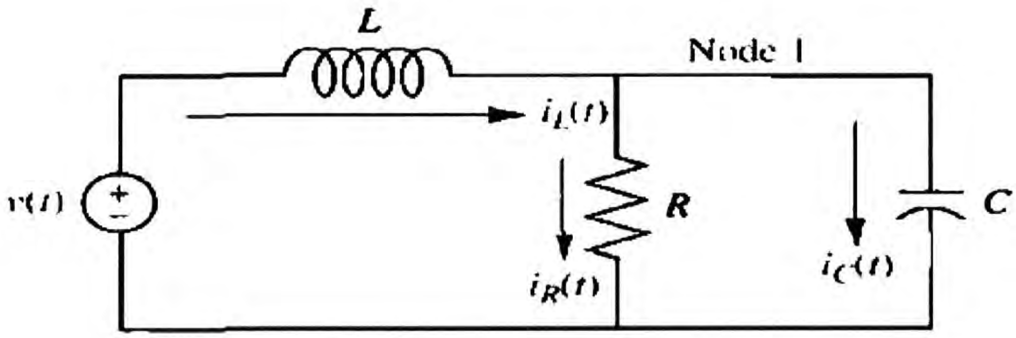

Figure 3

b) For the rotational mechanical system shown below in Figure 4, determine the transfer function $G(s) = \frac{Q_1(s)}{T(s)}$. Assume zero initial conditions. ***(8 marks)***

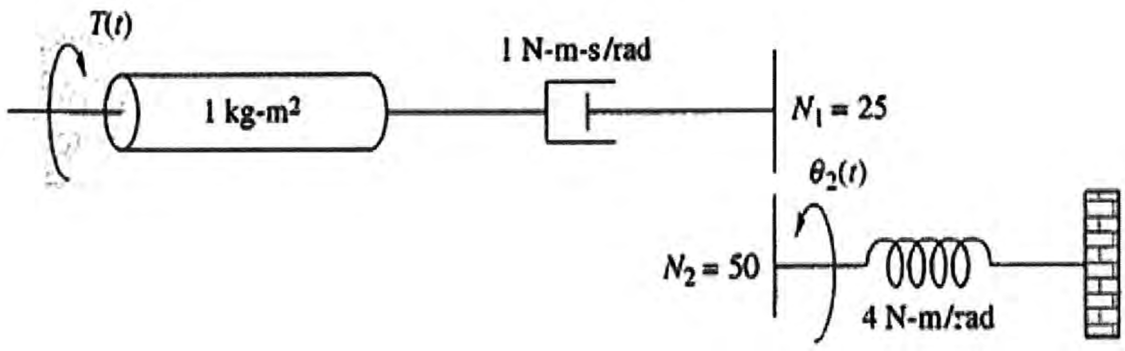

Figure 4

Question 3 (15 marks)

a) An optical encoder is used with a 10 cm diameter tracking wheel to measure linear displacement. The encoder generates 256 pulses per revolution (*NR*). Determine the total pulse count (*NT*) produced by the measurement of a linear displacement of 2 m. ***(3 marks)***

Please turn over

b) A bellows pressure element as shown in Figure 5 has the following values:
Effective area of bellows = 21 cm^2
Spring rate of the spring = 200 N/cm
Spring rate of the bellows = 10 N/cm
What is the pressure range of the transducer if the motion of the bellows is limited to 2 cm.
(3 marks)

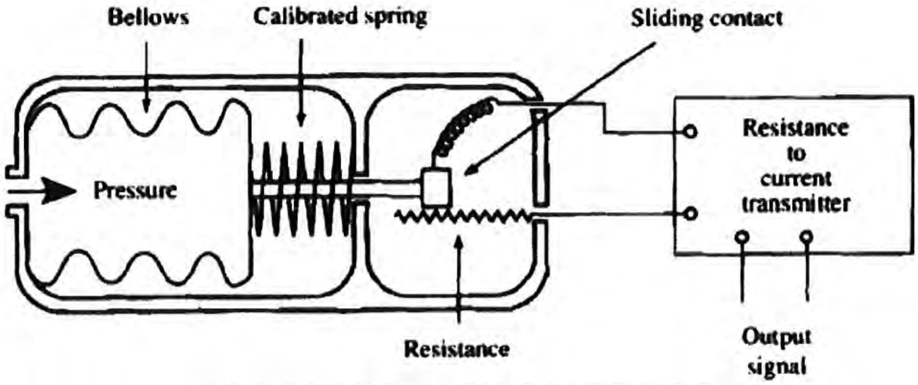

Figure 5: Bellows-resistance pressure sensor

c) A permanent magnet DC motor has armature resistance R_a of 0.05 ohm, armature inductance L_a of 5 mH and motor constant of 0.8 Nm/A. The motor and load have a combined moment of inertia J of 0.8 kgm^2 and damping coefficient B of 0.1 Nm per rad/sec. A 5 V step input was applied to the motor at time t = 0. Find the positional transfer function. Assume the motor was initially at rest. ***(5 marks)***

d) For the Pneumatic Actuator shown in Figure 6, determine the transfer function $X(s)/P(s)$, where $X(s)$ is the output displacement and $P(s)$ is the input pressure. ***(4 marks)***

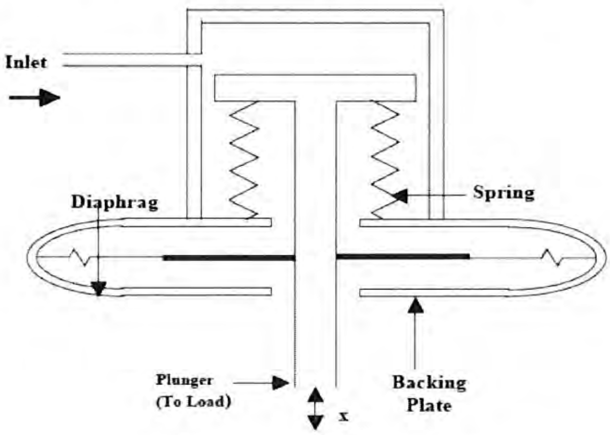

Figure 6 : Pneumatic Actuator

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Question 4 (13 marks)

a) For the unity feedback system of Figure 7:
   i) Determine the close loop transfer function, characteristic equation and the system type number. ***(5 marks)***
   ii) Use the Routh-Hurwitz test to determine the range of gain, k, for which the system is stable. ***(5 marks)***

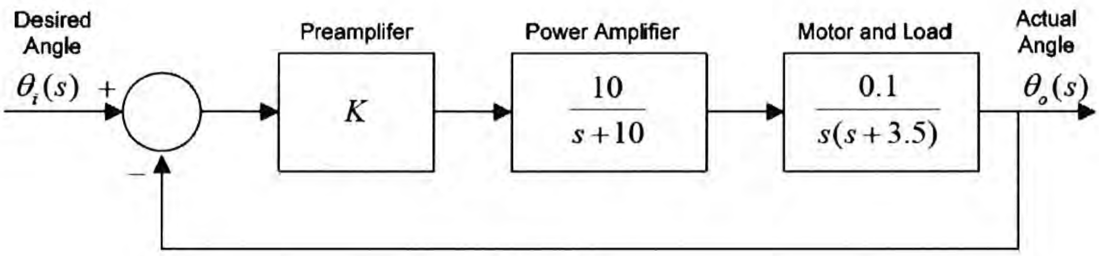

Figure 7

b) A second order feedback control system has characteristic equation given by:

$$1.5s^2 + 2.6s + 4 = 0$$

Find the damping coefficient ζ, the undamped natural frequency ω_n and the damped natural frequency ω_d of the system. ***(3 marks)***

Question 5 (17 marks)

a) Nyquist plot for a unity feedback system is given in the answer booklet. ***(4 marks)***
   i) Determine the gain margin and phase margin of the system.
   ii) Is the system stable?

b) For the open loop transfer function given by:

$$GH(s) = \frac{100000K(s+5)}{s(s+300)(s+10000)}$$

   i) Draw the Bode plot for the system with gain k equal to 1. ***(7 marks)***
   ii) Determine the gain margin and phase margin of the system. Is the system stable? ***(3 marks)***
   iii) Find the value of gain, *K,* to yield 9.5% overshoot in the transient response for a step input. ***(3 marks)***

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Question 6 (25 marks)

a) Figure 8 below shows an uncompensated unity feedback control system:

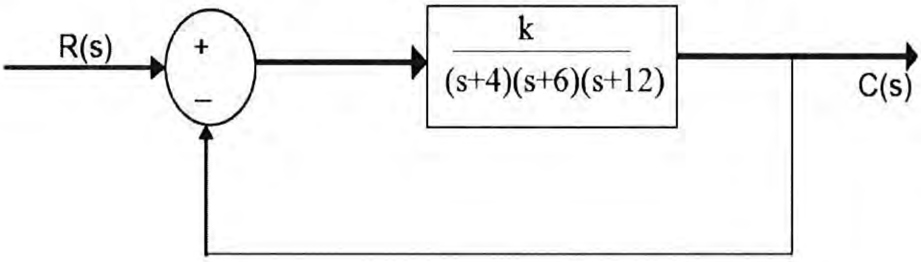

Figure 8: Uncompensated feedback control system

i) For the uncompensated feedback system of Figure 8 operating at 15% overshoot with step input, using root locus method compute the parameters listed in Table 1. ***(12 marks)***

Table 1: Parameters for uncompensated feedback system of Figure 8 at 15% overshoot

	Uncompensated system
Plant	$\frac{K}{(s+4)(s+6)(s+12)}$
Dominant poles	
K	
ζ	
w_n	
%OS	15
T_s	
T_p	
K_p	
$e(\infty)$	

ii) For the uncompensated feedback system of Figure 8, design a PD controller so that the system can operate at 15% overshoot with peak time that is two-third that of the uncompensated system. ***(8 marks)***

iii) For the uncompensated feedback system of Figure 8, design a PI controller so that the system can operate at 15% overshoot with zero steady-state error for a step input. ***(5 marks)***

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A: LAPLACE TRANSFORM OF COMMON FUNCTIONS

Time Functions f(t)	**Laplace Transform** $L\{f(t)\} = F(s)$
u(t), unit step. u(t) = 1	1/s
t, unit ramp	$1/s^2$
t^n	$n! / s^{n+1}$
e^{-at}	$1 / (s + a)$
$\cos \omega t$	$s / (s^2 + \omega^2)$
$\sin \omega t$	$\omega / (s^2 + \omega^2)$
$e^{-at} \cos \omega t$	$(s + a) / [(s + a)^2 + \omega^2]$
$e^{-at} \sin \omega t$	$\omega / [(s + a)^2 + \omega^2]$

B: LAPLACE TRANSFORM OPERATORS

Operation	**Time Domain**	**Laplace Domain**
Final Value theorem	$\lim_{t \to \infty} f(t)$	$\lim_{s \to 0} sF(s)$
Initial Value theorem	$\lim_{t \to 0} f(t)$	$\lim_{s \to \infty} sF(s)$
First Derivative	$\frac{d}{dt} f(t)$	$sF(s) - f(0)$
2nd Derivative	$\frac{d^2}{dt^2} f(t)$	$s^2F(s) - sf(0) - \frac{df(0)}{dt}$
nth Derivative	$\frac{d^n}{dt^n} f(t)$	$s^nF(s) - \sum_{r=1}^{n} \frac{d^{r-1}}{dt^{r-1}} f(0)\, s^{n-r}$
Complex Shift theorem	$e^{-at} f(t)$	$F(s + a)$
First Integral	$\int_0^t f(t)\, dt$	$(1/s)F(s)$
Multiplication by t	$t\, f(t)$	$-\frac{d}{ds} F(s)$
Division by t	$\frac{1}{t} f(t)$	$\int_s^\infty F(s) ds$

C: RELEVANT FORMULAE

System Performance & Specifications

1. $G(s) = \dfrac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$

2. $M_p = e^{-\left(\zeta\pi / \sqrt{1-\zeta^2}\right)}$

3. $\zeta = \sqrt{\dfrac{(\ln \frac{PO}{100})^2}{\pi^2 + (\ln \frac{PO}{100})^2}}$

4. $t_p = \dfrac{\pi}{\omega_n \sqrt{1-\zeta^2}} = \dfrac{\pi}{\omega_d}$

5. $T_s = \dfrac{4}{\zeta\omega_n}$

6. $\Phi_M = 90 - \tan^{-1}\dfrac{\sqrt{-2\zeta^2 + \sqrt{1+4\zeta^4}}}{2\zeta}$

   $= \tan^{-1}\dfrac{2\zeta}{\sqrt{-2\zeta^2 + \sqrt{1+4\zeta^4}}}$

D: Z -TRANSFORM

Function No	f(t) $t \geq 0$ f(t) = 0 for t < 0	Laplace Transform F(s) = L[f(t)]	f[k], f[kT] $k \geq 0$ f[kT] = 0 for k < 0	Z-Transform F(z) = Z{f[kT]}
1	Impulse function $\delta(t)$	1	$\delta[k]$	1
2	Unit step function u(t)	$\frac{1}{s}$	u[k]	$\frac{z}{z-1}=\frac{1}{1-z^{-1}}$
3	-		a^k	$\frac{z}{z-a}=\frac{1}{1-az^{-1}}$
4	-		ka^k	$\frac{az}{(z-a)^2}=\frac{az^{-1}}{(1-az^{-1})^2}$
5	t	$\frac{1}{s^2}$	kT	$\frac{Tz}{(z-1)^2}=\frac{Tz^{-1}}{(1-z^{-1})^2}$
6	t^2	$\frac{2}{s^3}$	$(kT)^2$	$\frac{T^2z(z+1)}{(z-1)^3}$
7	e^{-at}	$\frac{1}{(s+a)}$	e^{-akT}	$\frac{z}{z-e^{-aT}}=\frac{1}{1-z^{-1}e^{-aT}}$
8	$\sin(\omega t)$	$\frac{\omega}{(s^2+\omega^2)}$	$\sin[\omega kT]$	$\frac{z\sin(\omega T)}{z^2-2z\cos(\omega T)+1}$
9	$\cos(\omega t)$	$\frac{s}{(s^2+\omega^2)}$	$\cos[\omega kT]$	$\frac{z[z-\cos(\omega T)]}{z^2-2z\cos(\omega T)+1}$
10	te^{-at}	$\frac{1}{(s+a)^2}$	kTe^{-akT}	$\frac{Tze^{-aT}}{(z-e^{-aT})^2}=\frac{Tz^{-1}e^{-aT}}{(1-z^{-1}e^{-aT})^2}$
11	$e^{-at}\sin(\omega t)$	$\frac{\omega}{(s+a)^2+\omega^2}$	$e^{-akT}\sin[\omega kT]$	$\frac{ze^{-aT}\sin(\omega T)}{z^2-2ze^{-aT}\cos(\omega T)+e^{-2aT}}$
12	$e^{-at}\cos(\omega t)$	$\frac{(s+a)}{(s+a)^2+\omega^2}$	$e^{-akT}\cos[\omega kT]$	$\frac{z^2-ze^{-aT}\cos(\omega T)}{z^2-2ze^{-aT}\cos(\omega T)+e^{-2aT}}$

E: ASYMPTOTIC PLOTS ERRORS

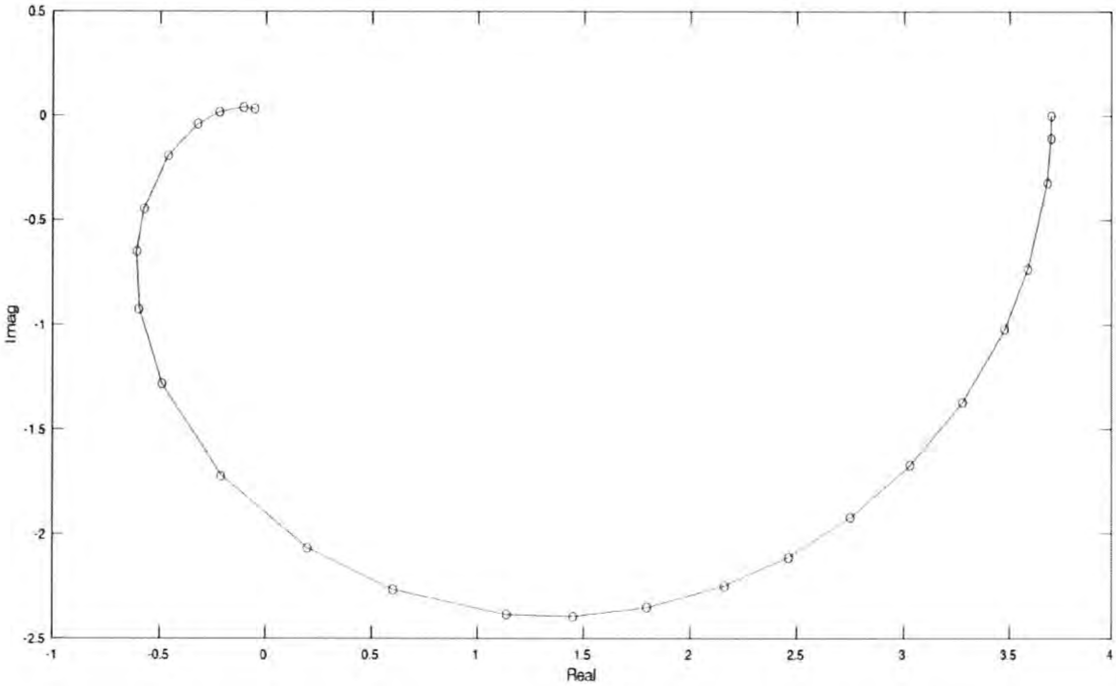
Asymptotic Plot Errors for $(1 + j\omega_i\tau_{zi})$

Corner Freq. $\omega_c = \frac{1}{\tau_z}$	$\frac{\omega_c}{5}$	$\frac{\omega_c}{2}$	ω	2ω	5ω
Mag. error dB	0.17	0.96	3.0	0.96	0.17
Phase angle error	11.3°	0.8°	0.0°	-0.8°	-11.3°

Asymptotic Plot Errors for $\frac{1}{1+j\omega_i\tau_{pi}}$

Corner Freq. $\omega_c = \frac{1}{\tau_z}$	$\frac{\omega_c}{5}$	$\frac{\omega_c}{2}$	ω	2ω	5ω
Mag. error dB	-0.17	-0.96	-3.0	-0.96	-0.17
Phase angle error	-11.3°	-0.8°	0.0°	0.8°	11.3°

EEB743 Answer Booklet for Question 5(a)



EEB743 Answer Booklet for Question 5(a)