



College of Engineering Science and Technology

School of Electrical and Electronics Engineering

Department of Electronics Engineering

Programme: Bachelor of Engineering (Telecommunication & Networking)

EEE694 -Engineering Mathematics 3

QUESTION PAPER
Final Examination: Semester 1, 2019

Instructions:

Read the following instructions before writing

1. Time Allowed: **3 hours** with additional **10 minutes** reading time.
2. All answers are to be written in the **Answer Booklet** provided.
3. Write your ID Number on all the sheets of paper of the Answer Booklet that you have used. Please also number all the sheets paper/pages that you use.
4. There are a total of 7 pages in the paper.
5. **There are a total of 12 questions. Answer any 10.**
6. Non-programmable calculators are allowed.

Question 1:

- a) The Table below describes all the mortgage loans made by a private Loan Company.

Thousands of dollars	Number of loans
0-20	3
20-40	5
40-60	6
60-80	4
80-100	2

Find the population mean, standard deviation, modal class and median class.

[1+2+1+1=5 Marks]

- b) If the light bulbs in a house fail according to a Poisson law, and over the last 15 weeks there have been 5 failures, find the probability that:

- i) there will be exactly two failures next week,
ii) there will not be more than one failure next week.

[2+3=5 Marks]

Question 2:

- a) A unbiased coin is tossed 3 times. Find

- i) The binomial probability that a head will occur exactly once.
ii) The expected (mean) number of heads.

[2+3=5 Marks]

- b) Wool fiber breaking strengths are normally distributed with mean $\mu = 23.56$ Newtons and standard deviation, $\sigma = 4.55$. What percentage of fibers would have a breaking strength of 14.45 or less?

[5 Marks]

Question 3:

A biologist assumes that there is a linear relationship between the amount of fertilizer supplied to tomato plants and the subsequent yield of tomato obtained.

Eight tomato plants of the same variety, were selected at random and treated, weekly, with a solution in which x grams of fertilizer was dissolved in a fixed quantity of water. The yield y kilograms of tomato were recorded.

Plant	A	B	C	D	E	F	G	H
Fertilizer x (in grams)	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5
Yield y (in kilograms)	3.9	4.4	5.8	6.6	7.0	7.1	7.3	7.7

- Find the correlation coefficient between x and y .
- Give a reason to support fitting a regression model of the form $y = a + bx$ to these data. Calculate the equation of the regression line of y on x .
- Estimate the yield of a plant treated, weekly, with 3.2 grams of fertilizer.
- Indicate why it may not be appropriate to use your equation to predict the yield of a plant treated, weekly with 20 grams of fertilizer. [3+4+2+1=10 Marks]

Question 4:

- Six Form IV students and six Form VI students were asked: How many books have you read this year? Here are their responses:
Form IV students: 5, 1, 2, 5, 3, 8
Form VI students: 4, 2, 0, 2, 3, 1
Calculate the sample mean and variance of each of these data sets. Which is more spread out, the Form IV or Form VI data set? [2+2+1=5 Marks]

- What values of (a, b, c) makes the function $f(x)$ a cubic spline?

$$f(x) = \begin{cases} x^3, & x \in [0, 1] \\ \frac{(x-1)^3}{2} + a(x-1)^2 + b(x-1) + c, & x \in [1, 3] \end{cases}$$

[5 Marks]

Question 5:

The operation manager of a large production plant would like to estimate the mean amount of time a worker takes to assemble a new electronic component. Assume that the standard deviation of this assembly time is 3.6 minutes.

- After observing 120 workers assembling similar devices, the manager noticed that their average time was 16.2 minutes. Construct a 95% confidence interval for the mean assembly time.
- How many workers should be involved in this study in order to have the mean assembly time estimated up to ± 15 seconds with 95% confidence?
- What happens to the confidence interval if you increase the confidence level?

[4+4+2=10 Marks]

Question 6:

- The observed values of a function at the four positions of the independent variable are given below.

x	3	7	9	10
$f(x)$	168	120	72	63

Use Newton's divided difference or Lagrange method to find the interpolating polynomial. What is the estimated value of the function at the position 6 of the independent variable? [4+1=5 Marks]

b) Find the maximum and minimum values of $f = 3x + 2y$ subject to constraints

$$x + 4y \leq 20,$$

$$2x + 3y \leq 30,$$

and non-negativity conditions

$$x \geq 0, y \geq 0.$$

[2.5+2.5=5 Marks]

Question 7:

Given the function $f(x) = x$, over the interval $-\pi < x < \pi$ and $f(x + 2\pi) = f(x)$.

- Sketch $f(x)$.
- Find whether $f(x)$ is even or odd function.
- Find the Fourier series expansion of $f(x)$.
- Hence show that

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$$

[1+1+6+2=10 Marks]

Question 8:

a) Check the orthogonality of the functions $y_m(x) = \sin(mx)$, $m = 1, 2, \dots$ on the interval $-\pi \leq x \leq \pi$. Also, obtain the orthonormal set of the functions.

[3+2=5 Marks]

b) Find the eigenvalues and eigenfunctions of the Sturm–Liouville problem.

$$y'' + \lambda y = 0, \quad \text{where } y(0) = 0, y(\pi) = 0.$$

[3+2=5 Marks]

Question 9:

$$\text{Let } f(x) = \begin{cases} \sqrt{2\pi}, & 0 < x < 2 \\ 0, & \text{otherwise.} \end{cases}$$

- Find the Fourier cosine and Fourier sine transforms of the function $f(x)$.
- Use the above result or otherwise, find the Fourier transform of the function $f(x)$.

$$\text{Hint: } \hat{f}(w) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x)e^{-iwx} dx.$$

[5+5=10 Marks]

Question 10:

a) Prove or disprove that $u(x, y) = e^x \cos(y)$ and $u(x, y) = \tan^{-1}\left(\frac{y}{x}\right)$ are solutions of the laplace's equation : $\nabla^2 u \equiv u_{xx} + u_{yy} = 0$. [2+3=5 Marks]

b) Use D'Alembert's method to solve one dimensional wave equation with $c^2 = 9$.

$$\begin{aligned}u_{tt} &= c^2 u_{xx}, 0 \leq x \leq 5, t \geq 0, \\u(0, t) &= 0 \text{ and } u(5, t) = 0, \\u(x, 0) &= f(x) = \sin(\pi x), \\u_t(x, 0) &= g(x) = 9x^2.\end{aligned}$$

[5 Marks]

Question 11:

a) Use the method of Characteristics to reduce the given PDE to its normal form and hence solve it.

$$u_{xx} + 2u_{xy} + u_{yy} = 0. \quad [5 \text{ Marks}]$$

b) The Fourier series representation for the heat equation $u_t = c^2 u_{xx}, 0 \leq x \leq L, t \geq 0$, $u(0, t) = 0$ and $u(L, t) = 0$, $u(x, 0) = f(x)$ is given by

$$u(x, t) = \sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi x}{L}\right) e^{-\lambda_n^2 t} \text{ where } \lambda_n = \frac{cn\pi}{L} \text{ and } B_n \text{'s are the Fourier sine series coefficients.}$$

Find the temperature in a laterally insulated copper bar ($c^2 = 1 \text{ cm}^2/\text{sec}$) 50 cm long if the initial temperature is $10 \sin\left(\frac{\pi x}{50}\right) \text{ }^\circ\text{C}$ and the ends are kept at $0 \text{ }^\circ\text{C}$. How long will it take for the maximum temperature in the bar to drop to $5 \text{ }^\circ\text{C}$?

[3+2=5 Marks]

Question 12:

a) Find $L\{e^{at} \sin(bt) \cos(ct) + t \sin(t)\}$ [5 Marks]

b) Use Laplace transform to solve

$$y'' + 8y' - 9y = t, \quad y(0) = 2, \quad y'(0) = 1.$$

[5 Marks]

Good Luck

THE END.

Table The Standard Normal Distribution

Cumulative Standard Normal Distribution

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
-3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
-3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
-0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
-0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
-0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
-0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
-0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
-0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
-0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
-0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
-0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
-0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

Table (continued)

Cumulative Standard Normal Distribution

<i>z</i>	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998