FIJI NATIONAL UNIVERSITY

COLLEGE OF ENGINEERING, SCIENCE AND TECHNOLOGY

School of Electrical & Electronics Engineering

Bachelor of Engineering (Honours) Electrical/Electronics Engineering

EEB731 – Signals and Systems

FINAL EXAMINATION

Semester 1, 2019

Date: As per Exam Time Table

Time: As per Exam Time Table

Venue: As per Exam Timetable

Duration of Exam: 3 Hours

Total Number of pages: 8

Instructions to Students

1. You are allowed an extra ten (10) minutes of reading time during which you are NOT allowed to write.
2. Attempt all questions.
3. Write your answers in the answer booklet provided.
4. Write your Student ID number on each page used.
5. Begin each Section on a fresh page and use both sides of the answer sheet.
6. You may use calculators provided they are non-programmable.
7. Clearly number the questions in your answer paper in their correct sequence and write legibly. Show all working.
8. Attach any extra sheets used to your answer booklet securely with the string provided.

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Question 1 (20 marks)

i) Show that the signal $x(t) = \exp(-j\omega_O t)$ is periodic with period $T = 2\pi / \omega_O$. Also, compute its power. ***(4 marks)***

ii) Consider the signal

$$x(t) = \begin{cases} 2t+3, & -3 \le t < -1 \\ t^2, & -1 \le t < 0 \\ 0, & otherwise \end{cases}$$

Plot the transformed signal y(*t*) = x(-2t+2) ***(8 marks)***

iii) A signal x(n) is defined by

$$x(n) = \begin{cases} 1, & 5 \le n \le 8 \\ 0, & otherwise \end{cases}$$

Represent the signal $x(n)$ in its delta form. ***(3 marks)***

iv) Evaluate the following integral:

$I_1 = \int_{-2}^{4} u(t-3)\delta(t-3)\, dt$ ***(2 marks)***

v) Two signals $g(n) = \delta(n-2)$ and $h(n) = \delta(n-1)$ are defined over the interval [0,4]. Determine if the signals are orthogonal. ***(3 marks)***

Question 2 (20 marks)

i) A discrete-time sequence $x(n)$ is defined as $x(n) = (½)^n\, u(n)$.

Sketch and determine the value of $x(n)\, \delta(n - 5)$. ***(3 marks)***

ii) Determine if the system described by $y(t) = x(t) + a$, is linear or non-linear, where a is constant. ***(4 marks)***

iii) Determine if the system defined by $y(t) = x(t)\sin(t+1)$ is time-invariant or not. ***(4 marks)***

iv) Plot the function x(t) for the time range $-8 \le t \le 8$. ***(4 marks)***

$$x(t) = u(t+4) - r(t-2) - \delta(t)u(t-4)$$

v) Consider the signals defined as follows:

$$h(n) = u(n) - u(n-4) \quad \text{and} \quad x(n) = n\, u(n)\, u(-n+3)$$

Compute the convolution of $h(n)$ and $x(n)$. ***(5 marks)***

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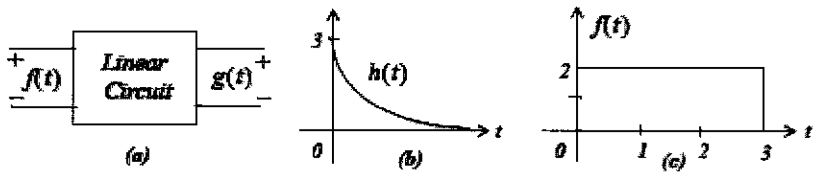
Question 3 (20 marks)

i) For the linear circuit in Figure below, the impulse response is shown in (b), where $h(t) = 3e^{-2t}$. Compute the response $g(t)$ for the input $f(t) = 2[u(t) - u(t-3)]$ using time-domain techniques. ***(10 marks)***



ii) As a design engineer of a company, you are required to demonstrate to the training staffs the equivalent of computing convolution in time domain. Using Laplace Transform, show that convolution in time domain is equivalent to multiplication in frequency domain. ***(5 marks)***

iii) Define and derive the equation for the average power of periodical signals using Parseval's theorem. ***(5 marks)***

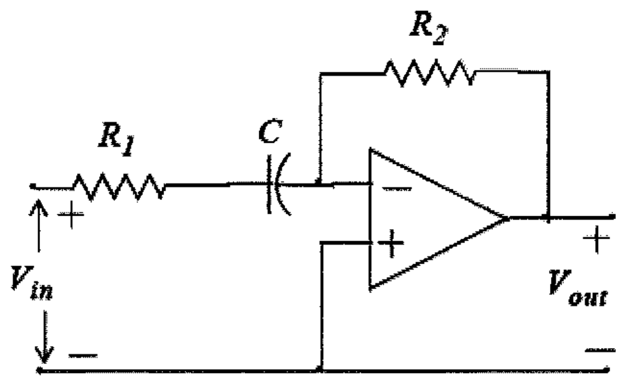
Question 4 (20 marks)

i) As a design engineer of a company you are to analyze signals coming from diverse disciplines and you are given the task to orthonormalize the following set of two signals: $v_1(t) = t^2$ and $v_2(t) = t^3$. Obtain the orthonormalized signals $u_1(t)$ and $u_2(t)$ for the time interval $-1 \leq t \leq 1$. ***(5 marks)***

ii) For the circuit system given in Figure below:

a) Determine the input-output relationship in the form of differential equation. ***(5 marks)***

b) Determine the impulse response $h(t)$ of the system using the Laplace Transformation technique. ***(5 marks)***



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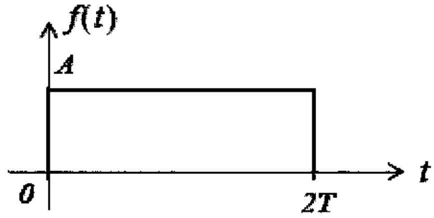
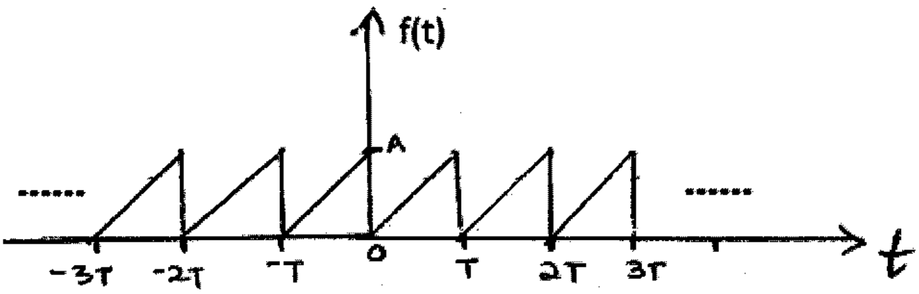
iii) Compute the Fourier Transform of the following pulse shown in Figure below.

(5 marks)



Question 5 (20 marks)

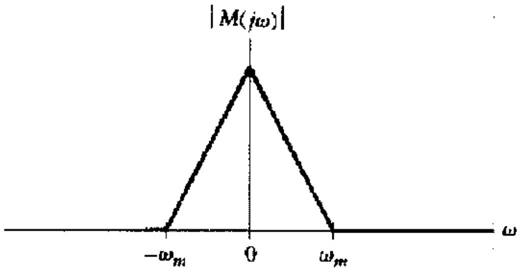
i) Consider the signal $f(t) = \frac{A}{T}t$ as shown below where $0 \le t \le T$, and $x(t) = x(t+T)$. Plot its magnitude and phase spectra. ***(10 marks)***



ii) Consider a message signal $m(t)$ with the spectrum shown below. The message bandwidth $w_m = 2\pi \times 10^3 rad/s$. This signal is applied to a product modulator, together with a carrier wave $A_c \cos(w_c\, t)$, producing the DSB-SC modulated signal $s(t)$. The modulated signal is next applied to a coherent detector. Assuming perfect synchronism between the carrier waves in the modulator and detector, determine the spectrum of the detector output when:

a) The carrier frequency $w_c = 2.5\pi \times 10^3 rad/s$
b) What is the lowest carrier frequency for which each component of the modulated signal $s(t)$ is uniquely determined.

(5 marks)



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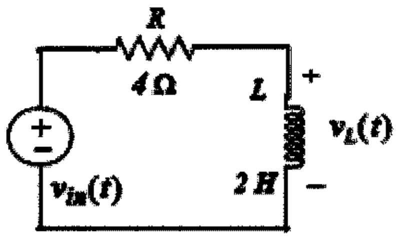
iii) For the circuit in Figure below, use Fourier Transform method and the system function $H(w)$ to compute $v_L(t)$. Assume zero initial conditions and $V_{in}(t) = 5e^{-3t}u(t)$. ***(5 marks)***



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Fourier Transform Table

Time Signal	Fourier Transform
$1, \quad -\infty < t < \infty$	$2\pi\delta(\omega)$
$-0.5 + u(t)$	$1/j\omega$
$u(t)$	$\pi\delta(\omega) + 1/j\omega$
$\delta(t)$	$1, \quad -\infty < \omega < \infty$
$\delta(t-c), \quad c$ real	$e^{-j\omega c}, \quad c$ real
$e^{-bt}u(t), \quad b > 0$	$\dfrac{1}{j\omega + b}, \quad b > 0$
$e^{j\omega_o t}, \quad \omega_o$ real	$2\pi\delta(\omega - \omega_o), \quad \omega_o$ real
$p_\tau(t)$	$\tau \operatorname{sinc}[\tau\omega/2\pi]$
$\tau \operatorname{sinc}[\tau t/2\pi]$	$2\pi p_\tau(\omega)$
$\left[1 - \frac{2\|t\|}{\tau}\right] p_\tau(t)$	$\frac{\tau}{2}\operatorname{sinc}^2[\tau\omega/4\pi]$
$\frac{\tau}{2}\operatorname{sinc}^2[\tau t/4\pi]$	$2\pi\left[1 - \frac{2\|\omega\|}{\tau}\right] p_\tau(\omega)$
$\cos(\omega_o t)$	$\pi[\delta(\omega + \omega_o) + \delta(\omega - \omega_o)]$
$\cos(\omega_o t + \theta)$	$\pi[e^{-j\theta}\delta(\omega + \omega_o) + e^{j\theta}\delta(\omega - \omega_o)]$
$\sin(\omega_o t)$	$j\pi[\delta(\omega + \omega_o) - \delta(\omega - \omega_o)]$
$\sin(\omega_o t + \theta)$	$j\pi[e^{-j\theta}\delta(\omega + \omega_o) - e^{j\theta}\delta(\omega - \omega_o)]$

Fourier Transform Properties

Property Name	Property	
Linearity	$ax(t)+bv(t)$	$aX(\omega)+bV(\omega)$
Time Shift	$x(t-c)$	$e^{-j\omega c}X(\omega)$
Time Scaling	$x(at), \quad a \neq 0$	$\frac{1}{a}X(\omega/a), \quad a \neq 0$
Time Reversal	$x(-t)$	$X(-\omega)$ $\overline{X(\omega)}$ if $x(t)$ is real
Multiply by t^n	$t^n x(t), \quad n=1,2,3,\ldots$	$j^n \frac{d^n}{d\omega^n}X(\omega), \quad n=1,2,3,\ldots$
Multiply by Complex Exponential	$e^{j\omega_o t}x(t), \quad \omega_o$ real	$X(\omega-\omega_o), \quad \omega_o$ real
Multiply by Sine	$\sin(\omega_o t)x(t)$	$\frac{j}{2}[X(\omega+\omega_o)-X(\omega-\omega_o)]$
Multiply by Cosine	$\cos(\omega_o t)x(t)$	$\frac{1}{2}[X(\omega+\omega_o)+X(\omega-\omega_o)]$
Time Differentiation	$\frac{d^n}{dt^n}x(t), \quad n=1,2,3,\ldots$	$(j\omega)^n X(\omega), \quad n=1,2,3,\ldots$
Time Integration	$\int_{-\infty}^{t} x(\lambda)d\lambda$	$\frac{1}{j\omega}X(\omega)+\pi X(0)\delta(\omega)$
Convolution in Time	$x(t)*h(t)$	$X(\omega)H(\omega)$
Multiplication in Time	$x(t)w(t)$	$\frac{1}{2\pi}X(\omega)*W(\omega)$
Parseval's Theorem (General)	$\int_{-\infty}^{\infty} x(t)\overline{v(t)}dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)\overline{V(\omega)}d\omega$	
Parseval's Theorem (Energy)	$\int_{-\infty}^{\infty} x^2(t)dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} \lvert X(\omega)\rvert^2 d\omega$ if $x(t)$ is real $\int_{-\infty}^{\infty} \lvert x(t)\rvert^2 dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} \lvert X(\omega)\rvert^2 d\omega$	
Duality: If $x(t) \leftrightarrow X(\omega)$	$X(t)$	$2\pi x(-\omega)$

Table 1.1: LAPLACE TRANSFORM OF COMMON FUNCTIONS

Row No	**Time Function f(t)**	**Laplace Transform** L {f(t)} = F(s)
1	u(t) , unit step. u(t) = 1	1/s
2	t, unit ramp	$1/s^2$
3	t^n	$n! / s^{n+1}$
4	e^{-at}	1 / (s + a)
5	Cos ωt	$s / (s^2 + \omega^2)$
6	Sin ωt	$\omega / (s^2 + \omega^2)$
7	$e^{-at} \cos \omega t$	$(s + a) / [(s + a)^2 + \omega^2]$
8	$e^{-at} \sin \omega t$	$\omega / [(s + a)^2 + \omega^2]$

TABLE 1.2: LAPLACE TRANSFORM OF OPERATIONS (for causal system)

Row No	**Operation**	**Time Domain**	**Laplace domain**
1	Final Value theorem	$\lim_{t->\infty} f(t)$	$\lim_{s->0} sF(s)$
2	Initial Value Theorem	$\lim_{t->0} f(t)$	$\lim_{s->\infty} sF(s)$
3	First Derivative	$\frac{d}{dt} f(t)$	$sF(s) – f(0)$
4	2^{nd} Derivative	$\frac{d^2}{dt^2} f(t)$	$s^2F(s) - sf(0) - \frac{df(0)}{dt}$
5	n^{th} Derivative	$\frac{d^n}{dt^n} f(t)$	$s^nF(s) - \sum_{r=1}^{n} \frac{d^{r-1}}{dt^{r-1}} f(0)\, s^{n-r}$
6	Complex Shift Theorem	$e^{-at}f(t)$	F(s + a)
7	First Integral	$\int_0^t f(t)dt$	(1/s)F(s)
8	Scaling Theorem	f(t/a)	aF(as)
9	Multiplication by constant	kf(t)	kF(s)
10	Multiplication by t	tf(t)	$-\frac{d}{ds} F(s)$
11	Division by t	$\frac{1}{t} f(t)$	$\int_s^\infty F(s)ds$