FNU FIJI NATIONAL UNIVERSITY

College of Engineering, Science and Technology

School of Electrical &Electronics Engineering

EEE 694 – Engineering Mathematics III

Semester 2, 2018

SUPPLEMENTARY EXAMINATION

Programme: ***Bachelor of Engineering***

Time Allowed: ***3 HOURS 10 MINS***

Time: **Date:**

Instructions to Students:

1. There are thirteen (13) questions in the paper. Answer any ten (10) questions.
2. You are allowed 10 minutes extra reading time during which you are NOT allowed to write.
3. This exam is worth 50% of your overall marks.
4. Answer questions neatly on a new page and clearly number the question attempted.
5. Students may use a calculator, provided it is silent and non-programmable.
6. If you use extra sheets of paper, attach it securely to the answer booklet.

(1) Find the charge and the current for $t > 0$ in a series RC circuit where $R = 10\Omega$, $C = 4 \times 10^{-3}F$ and $E = 85cos150t\ V$. Assume that when the switch is closed at $t = 0$, the charge on the capacitor is $-0.005C$.

[10]

(2) (a) Verify by substitution that the functions $y = x^2$ and $y = 1$ are solutions of the nonhomogeneous linear ordinary differential equation $y''y - xy' = 0$, but their sum is a not a solution.

(b) Solve the ordinary differential equation $2xyy' = y^2 - x^2$

[6+4]

(3) (a) Solve the initial value problem $(\text{cosy sinhx} + 1)dx - \sin y\ coshx\ dy = 0$, y(1)=2

(b) Solve the initial value problem $y' + ytanx = sin2x, y(0) = 1$

[4+6]

(4) (a) Solve the initial value problem $y'' + y' - 2y = 0, y(0) = 4, y'(0) = -5$.

(b) Verify and explain why $y = e^{-2x}$ is a solution of $y'' - y' - 6y = 0$ but xe^{-2x} is not.

[5+5]

(5) (a) Solve the linear ordinary differential equation: $xy' = 2y + e^x x^3$.

(b) Let the electric equipotential lines between two concentric cylinders with the z-axis in space be given by $u(x, y) = x^2 + y^2 = c$. Find their orthogonal trajectories (the curves of electric force).

[5+5]

(6) (a) Solve the first order ordinary differential equation $(x^2 + y^2)dx - 2xydy = 0$.

(b) Test whether the following equation is an exact differetial equation. If so, solve

$$\cos(x + y)\, dx + (3y^2 + 2y + \cos(x + y)\, dy = 0.$$

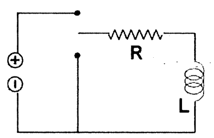
[5+5]

(7) (a) Let T_1 and T_2 be linear transformation from R^3 into R^3 such that $T_1(x, y, z) = (-2x + y, -x , x + 3z)$ and $T_2(x, y, z) = (x - 3y, 4z, x + 7y)$. Find the standard matrices for the compositions $T = T_1 o\, T_2$ and $T' = T_2 o\, T_1$

(b) Find the standard matrix for the linear transformation $T: R^3 \to R^2$ defined by $T(x, y, z) = (3x - 5y, 2x + 7y)$

[6+4]

(8) Suppose that in the simple circuit as shown in the figure below the resistance is 10 Ω and the inductance is 5H. If a battery gives a constant voltage of 75V and the switch is closed when t = 0 so the current starts with I(0) = 0, find:



(i) current I(t),

(ii) the current after 5 sec, and

(iii) the limiting value of the current.

[5+2+3]

(9) Show that the matrix $A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 2 & 1 \\ 1 & 0 & 2 \end{pmatrix}$ is diagonalizable. Then find a matrix P such that $P^{-1}AP$ is diagonal.

[10]

(10) Matrix A is given as $A = \begin{pmatrix} 3 & 2 & -3 \\ -3 & -4 & 9 \\ -1 & -2 & 5 \end{pmatrix}$. Find

(i) the characteristic equation;

(ii) the Eigenvalues and corresponding Eigen vectors of the matrix A.

[10]

(11) (a) A series of RL circuit with $R = 50\Omega$ and L=10H has a constant voltage $V = 100V$ applied at $t = 0$ by the closing of a switch. Find the equation for current I(t).

(b) Find the standard matrix for the linear operator T , where

$$T(x_1, x_2, x_3) = (x_1 + 2x_2 + x_3, x_1 + 5x_2, x_3).$$

[5+5]

(12) Find the current I(t) in an RLC-circuit with R = 2 Ω , L =0.5H , C = 0.005 F , which is connected to a source of EMF E(t) = sin t Volts (assuming zero initial current and charge).

[10]

(13) The linear transformation T is given by T(x) = Ax, where $A = \begin{pmatrix} 2 & 4 & 8 & 16 \\ 16 & 8 & 4 & 2 \\ 4 & 8 & 16 & 2 \\ 2 & 16 & 8 & 4 \end{pmatrix}$

Find (i) rank (T)

(ii) Nullity (T)

[10]

-THE END-