FIJI NATIONAL UNIVERSITY

COLLEGE OF ENGINEERING, SCIENCE & TECHNOLOGY

SCHOOL OF MATHEMATICAL & COMPUTING SCIENCES

DEPARTMENT OF MATHEMATICS & STATISTICS

PROGRAMME: ADVANCED DIPLOMA IN ELECTRONICS & ELECTRICAL ENGINEERING

EEE605 – MATHEMATICS FOR ENGINEERS

FINAL EXAMINATION

SEMESTER 1, 2018

Time Allocated: 3 hours
(plus extra 10 minutes reading time)

Total Marks: 100

Instructions

1. There are a total of four pages in this question paper, containing twelve questions, each worth 10 marks. Do ANY TEN questions.
2. Show all working as partly correct answers may be rewarded.
3. Programmable calculators are not allowed.
4. This exam is worth 50% of your overall mark.
5. If you use extra sheets of paper, attach it securely to the Answer Booklet.

QUESTION 1

a) Use **Simpson's rule** with $n=4$ to approximate the value of $\int_2^4 \sqrt{1+x^3}\,dx$. [5m]

b) Using **Euler's method** with step size $h=0.1$, approximate the value of the solution to the initial value problem $y'=2y-x$, $y(2)=3$ at $x=2.5$. [5m]

QUESTION 2

Find the **particular solution** to the initial value problem $x^3y'-6x^4y=x^3e^{3x^2}$, where $y(0)=2$. [10m]

QUESTION 3

a) Find the **solution** of the homogeneous ODE $y''+y'-2y=0$. [4m]

b) Use the method of **undetermined coefficients** to find a **base solution** of the ODE $y''+y'-2y=5e^{2x}$. [4m]

c) Write out the **general solution** of the ODE $y''+y'-2y=5e^{2x}$. [2m]

QUESTION 4

a) Evaluate the iterated integral $\int_1^3\int_{-1}^1 (4y-2x)\,dy\,dx$. [5m]

b) Evaluate the triple integral $\iiint_G 12xy^2z^3\,dV$ over the rectangular box G defined by the inequalities $-1\le x\le 2, 0\le y\le 3, 0\le z\le 2$. [5m]

QUESTION 5

Given are two vectors $u=<3,-2,1>$ and $v=<2,4,-1>$.

a) Find the scalar product of ***u*** and *v*. [2m]

b) Find the angle between ***u*** and *v*. [3m]

c) Find the cross product of ***u*** and *v*. [5m]

QUESTION 6

a) Find the derivative of $r(t) = (1+t^3)\ \hat{i} + 3te^{-t}\ \hat{j} + \sin 3t\ \hat{k}$ and find the unit tangent vector at the point where $t = 0$. [4m]

b) A moving particle starts at an initial position $r(0) = \langle 1,0,0 \rangle$ with an initial velocity of $v(0) = \hat{i} - \hat{j} + \hat{k}$. Its acceleration is $a(t) = 3t\ \hat{i} - 2t\ \hat{j} + \hat{k}$. Find its velocity equation and position equation at time t. [6m]

QUESTION 7

a) Find all the **critical points** of the function $f(x,y) = 2x^2 - 4xy + y^4 + 2$. [5m]

b) Use the **second derivative** test to classify each **critical point** found in part (a) as either a **local minimum, local maximum**, or a **saddle point**. [5m]

QUESTION 8

If $F(x,y,z) = xz^3\ \hat{i} + 2y^4x^2\ \hat{j} + 5z^2y\ \hat{k}$ then find:

a) div **F**. [4m]

b) curl **F**. [6m]

QUESTION 9

a) Compute the **line integral** $\int_C 3x^2yz\ ds$ along the curve $x = t,\ y = t^2,\ z = \frac{2}{3}t^3 \quad (0 \le t \le 1)$. [5m]

b) Evaluate the **line integral** $\int_C (3x^2 + y^2)\ dx + 2xy\ dy$ along the circular arc given by $x = \cos t,\ y = \sin t\ (0 \le t \le \frac{\pi}{2})$. [5m]

QUESTION 10

a) Find the **directional derivative** of the function $f(x,y) = e^{3xy}$ at the point $(2,0)$ in the direction of $u = -\frac{2}{5}\ \hat{i} + \frac{3}{5}\ \hat{j}$. [5m]

b) Find the **least squares regression line** for the points $\{(1,0),(2,2),(4,5)\}$. [5m]

QUESTION 11

a) Compute the **rank** and **nullity** of:

$$\begin{bmatrix} 1 & 2 & -2 & 1 \\ 3 & 6 & -5 & 4 \\ 1 & 2 & 0 & 3 \end{bmatrix}.$$

[4m]

b) Find all the **eigenvalues** and **eigenvectors** of the matrix $A = \begin{bmatrix} 7 & 3 \\ 3 & -1 \end{bmatrix}$. [6m]

QUESTION 12

Determine if the following system of linear equations given below has a solution and, in the case it does, determine all solutions.

$$\begin{aligned} x + 3y - 2z &= 5 \\ 3x + 5y + 6z &= 7 \\ 2x + 4y + 3z &= 8 \end{aligned}$$

[10m]

THE END