FIJI NATIONAL UNIVERSITY
College of Engineering, Science & Technology

SCHOOL OF ELECTRICAL & ELECTRONICS ENGINEERING

BACHELOR OF ENGINEERING

EEE701 – Fields and Waves

Supplementary Examination
TRIMESTER 1, 2017

DAY/DATE: As timetabled DURATION : Three hours
ROOM: As timetabled

INSTRUCTION TO STUDENTS

1. You are allowed 10 minutes extra reading time during which you are **NOT** to write.
2. **Begin the answer to each Question** on a fresh page and use both sides of the sheet.
3. Write clearly the number of the question attempted on the top of each sheet
4. Write your candidate number at the top of each sheet & attach them.
5. Insert all written foolscaps, graph paper etc. in their correct sequence and secure with a string.
6. All sheets of paper on which rough/draft work has been done, cross it through and attach all of them to your answer scripts.
7. Where ever possible, draw clear neat diagrams
8. Some useful mathematical relations are given in page 5

Number of pages including instruction page = 5

PTO

Useful constants: $\varepsilon_o = 8.854 \times 10^{-12}\ \text{Fm}^{-1}$; $\left|\frac{1}{4\pi\varepsilon_0}\right| = 9\times 10^9$; $\mu_o = 4\pi \times 10^{-7}\ \text{Hm}^{-1}$

Answer **ALL** questions

Question 1

i) Draw a Cartesian coordinate system and indicate the point Q (2, -2, -1). Write and indicate the position vector of the point Q.

(3 marks)

ii). Indicate points $M(-1, 2, 1)$, $N(3, -3, 0)$ in a Cartesian coordinate system. Indicate and find: *i*) the vector MN, *ii*). The distance MN

(4 marks)

iii) State the Stokes theorem in vector calculus as an equation. Define all the quantities used in the equation

(3 marks)

iv) Describing all the symbols used, and indicating them on a diagram, express in mathematical forms the Gauss' law in electrostatics.

(4 marks)

v) Giving a labelled diagram and stating the quantities used, write down the Biot-Sarvats law in magnetism as a mathematical equation. The directions of all vectors should be clearly indicated.

(4 marks)

vi) In a cross sectional view of a coaxial cable, draw and label the direction of flow of waves and the electric and magnetic components of the fields travelling as electromagnetic waves

(2 marks)

Question 2

A spherical charge distribution, whose centre lies at the origin of a Cartesian coordinate system and of radius 0.5 m as shown in Fig. Q2, carries a charge of density 40 nC m^{-3}.

Copy this diagram and indicate the quantities that represent point P in spherical coordinates. Using the Gauss theorem or otherwise, calculate the electric field in spherical coordinates at point P(0, 3m, 2m) due to the charge distribution.

(4 marks)

Transform this field into Cartesian coordinates.

(9 marks)

A point charge of 18 nC is now kept at R (0, 0, 2m) as shown in the Fig.Q2. Calculate the total electric field at P.

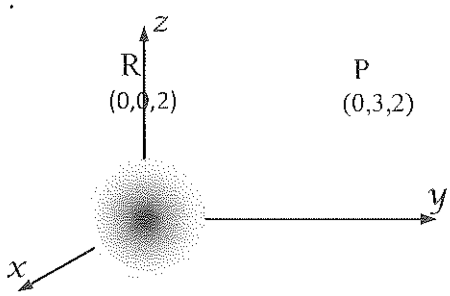

Fig. Q2

(7 marks)

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Question 3

In spherical coordinates, the electric field (**E**) at a point due to a spherical shell is given by $\mathbf{E} = \frac{24}{r^2}\,\mathbf{r}$. The radius of the shell is 0.4 m.

a) Derive, in spherical coordinates, an expression for the potential (V) at any point outside the shell, and calculate the potential at 0.5 m from the centre of the shell (6 marks)

b) If the radius of the shell is 0.4 m, calculate the potential of the shell. (2 marks)

c) Explaining the reasons, calculate the potential at the centre of the shell. (6 marks)

d) Giving some of the magnitudes of E and V, plot their (**E** and V) variation with the radial distance *r* from the centre. (6 marks)

Question 4

a) Starting with the Bio –Savart's law in magnetism, it can be shown that the magnetic induction field **B** at a point P, which is at a distance ρ_0 away from a a current carrying conductor is $\mathbf{B} = \frac{\mu_0 I}{4\pi \rho_0}(\cos\alpha_2 - \cos\alpha_1)$ where α_1 is the angle at the current entering end and α_2 is the angle at the current leaving end as shown in Fig. Q4(*a*).

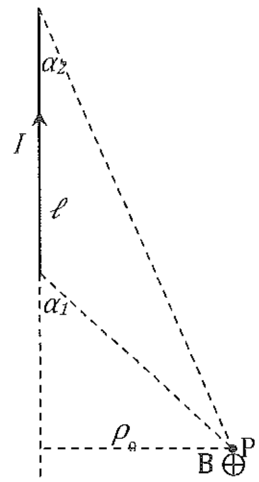

Fig. Q4 (*a*)

i) If the current I = 5 A, and the length of the conductor ℓ = 4m, calculate the induction field **B** at a point 2.5m away and along the perpendicular bisector to the length of the conductor.
(***Hint***: *Draw a diagram and mark the angles*) (4 marks)

ii) A square loop of wire of sides 4 m length carries a current of 5 A. Draw a neat diagram and indicate the direction of the current in the loop. (3 marks)

iii) Calculate the induction field **B** at the centre of the loop. Comment on the direction of **B**. (4 marks)

b) i) Describing the quantities used, state the Amperes circuital law and the Amperes force law in mathematical forms. (3 marks)

ii) Two long current carrying conductors carrying currents 10A and 15A in opposite directions are kept in air as shown in Fig Q4(*b*). The separation between the conductors is 0.8 m. Calculate the force per unit length in each conductor. Copy the diagram and mark the direction of the forces (6 marks)

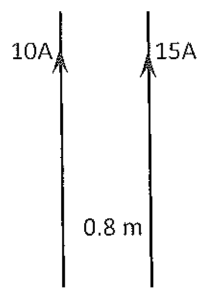

Fig Q4(*b*)

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Question 5

a) The voltage and current waves on a transmission line are given by

$V = \left(A\, e^{-\beta x} + B\, e^{\beta x}\right)e^{j\omega t}$ and $I = \frac{1}{Z_0}\left(A\, e^{-\beta x} - B\, e^{\beta x}\right)e^{j\omega t}$ where all the symbols have their usual meanings.

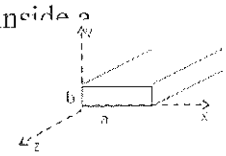
i) Show that if a load Z is connected to the transmission line, the voltage reflection coefficient B/A is: $\frac{B}{A} = \frac{Z_0 - Z}{Z_0 + Z}$

(4 marks)

ii) A transmission line of characteristic impedance 75 Ω feeds a 30 Ω load. If the input signal is $5\, e^{j\, 5\times 10^6\, t}$, write down the expression for the reflected signal and draw the incident and the reflected waves on the transmission line.

(6 marks)

b) The cross-section of a wave guide is shown in the figure. For EM waves inside a wave guide, the propagation coefficient P is given by:



$$P^2 = \left(\frac{m\,\pi}{a}\right)^2 + \left(\frac{n\,\pi}{b}\right)^2 - \varepsilon_0\varepsilon_r\mu_0\mu_r\omega^2 \text{ and } P = \alpha + j\,\beta$$

Where all the symbols have their usual meanings

The cross sectional dimension of the waveguide used in our laboratory is 8mm × 15 mm. Calculate the lowest frequency of the wave that can propagate in an air filled wave guide

(5 marks)

c) For an optical fiber, the refractive indices of the "*fiber*" and the "*cladding*" are 1.5 and 1.35 respectively. Deriving any equations used calculate the numerical aperture of the fiber.

(5 marks)

THE END

Some useful mathematical relations:

Relationship between different sets of coordinates:

	Cartesian x, y, z	Cylindrical ρ, ϕ, z	Spherical r, θ, ϕ
Cartesian x, y, z		$x = \rho \cos \phi$ $y = \rho \sin \phi$ $z = z$	$x = r \sin \theta \cos \phi$ $y = r \sin \theta \sin \phi$ $z = r \cos \theta$
Cylindrical ρ, ϕ, z	$\rho = \sqrt{x^2 + y^2}$ $\phi = \tan^{-1} \frac{y}{x}$ $z = z$	$\rho = r \sin \theta$ $\phi = \phi$ $z = r \cos \theta$	
Spherical r, θ, ϕ	$r = \sqrt{x^2 + y^2 + z^2}$ $\theta = \tan^{-1} \frac{\sqrt{x^2 + y^2}}{z}$ $\phi = \tan^{-1} \frac{y}{x}$	$r = \sqrt{\rho^2 + z^2}$ $\theta = \tan^{-1} \frac{\rho}{z}$ $\phi = \phi$	

To convert between different coordinates systems:

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix}$$

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi \\ \sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi \\ \cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix}$$