FNU FIJI NATIONAL UNIVERSITY

COLLEGE OF ENGINEERING, SCIENCE AND TECHNOLOGY

School of Electrical & Electronics Engineering

Bachelor of Engineering

EEE743 – Control Systems

FINAL EXAMINATION

Semester 1, 2017

Date: As per Exam Time Table

Time: As per Exam Time Table

Venue: As per Exam Timetable

Duration of Exam: 3 Hours 10 Minutes

Instructions to Students

1. You are allowed an extra ten (10) minutes of reading time during which you are NOT allowed to write.
2. Questions 1 to 6 are compulsory and select any one question from questions 7 and 8.
3. Write your answers in the answer booklet provided.
4. Write your Student ID number on each page used.
5. Begin each Section on a fresh page and use both sides of the answer sheet.
6. You may use calculators provided they are non-programmable.
7. Clearly number the questions in your answer paper in their correct sequence and write legibly. Show all working.
8. Attach any extra sheets used to your answer booklet securely with the string provided.

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Question 1

a) Find the transfer function $\frac{V_o(s)}{I_s(s)}$ of the system of Figure 1. Assume zero initial conditions with C=0.1F, L=1H and R=10Ω. ***(6 marks)***

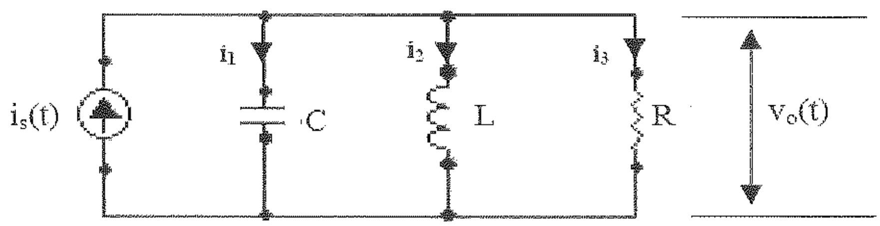

Figure 1: Electrical system

b) Determine the time domain output response for the system of (a), when a ramp input of magnitude 2 is applied. ***(9 marks)***

Question 2

Reduce the block diagram of Figure 2(a) to obtain a unity feedback system of Figure 2(b): compute the transfer function G(s). ***(5 marks)***

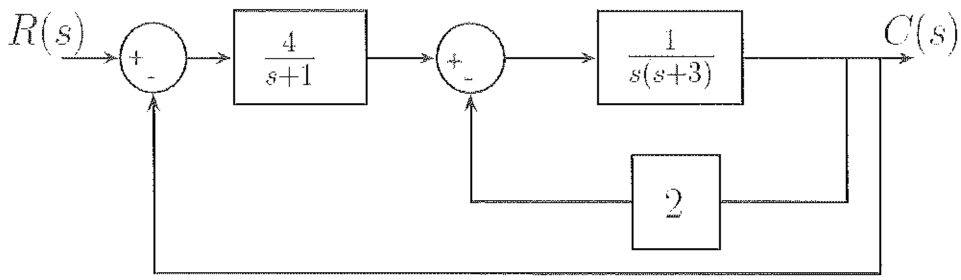

Figure 2 (a) Block diagram

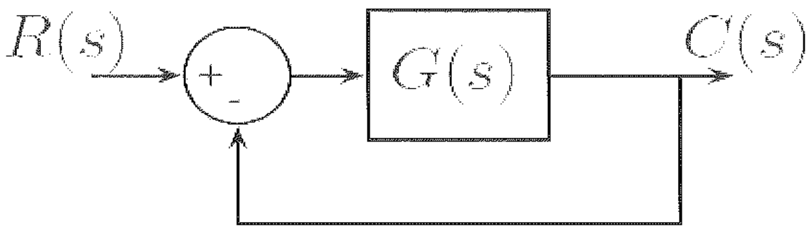

Figure 2 (b): Reduced Block diagram

Please Turn Over

Question 3

a) For the unity feedback system of Figure 3:
   i) Determine the close loop transfer function, characteristic equation and the system type number. ***(5 marks)***
   ii) Use the Routh-Hurwitz test to determine the range of gain, k, for which the system is stable. ***(10 marks)***

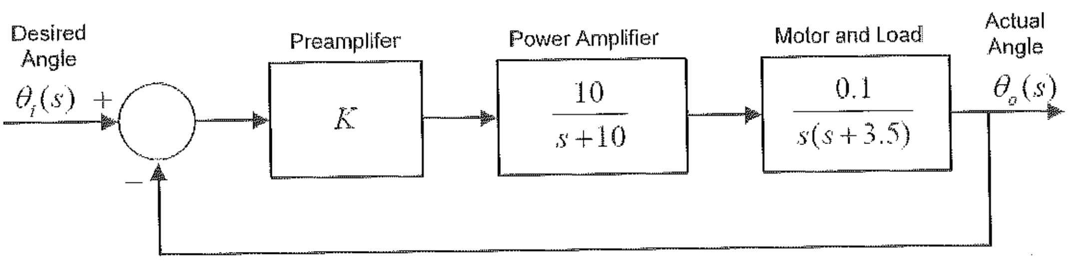

Figure 3

b) A second order feedback control system has characteristic equation given by:

$$1.5s^2 + 2.6s + 4 = 0$$

Find the damping coefficient ζ, the undamped natural frequency ω_n and the damped natural frequency ω_d of the system. ***(5 marks)***

Question 4

a) Nyquist plot for a unity feedback system is given in the answer booklet. ***(5 marks)***
   i) Determine the gain margin and phase margin of the system.
   ii) Is the system stable? Provide justification for your answer.

b) For the open loop transfer function given by:

$$GH(s) = \frac{100000K(s+5)}{s(s+300)(s+10000)}$$

   i) Draw the Bode plot for the system with gain k equal to 1. ***(10 marks)***
   ii) Determine the gain margin and phase margin of the system. Is the system stable? ***(5 marks)***
   iii) Find the value of gain, K, to yield 9.5% overshoot in the transient response for a step input. (only use bode plot of (i)) ***(5 marks)***

Please Turn Over

Question 5

Given plant transfer function as $G_1(s) = \frac{s+3}{s+2}$ and zero-order hold (z.o.h) transfer function as $G_h(s) = \frac{1-e^{-Ts}}{s}$, cascade z.o.h with $G_1(s)$ and find the resulting sampled-data transfer function as $G(z)$, if the samling time T = 0.5 second. ***(5 marks)***

Question 6

Figure 4(a) given below shows a control system with PID controller designed for closed loop operation. In practice, a PID controller can be realized using the circuit of operational amplifier as shown in Figure 4(b). Assuming C2=0.5F, select values of R1 , R2 and C1 to realize PID transfer function of Figure 4(a). ***(5 marks)***

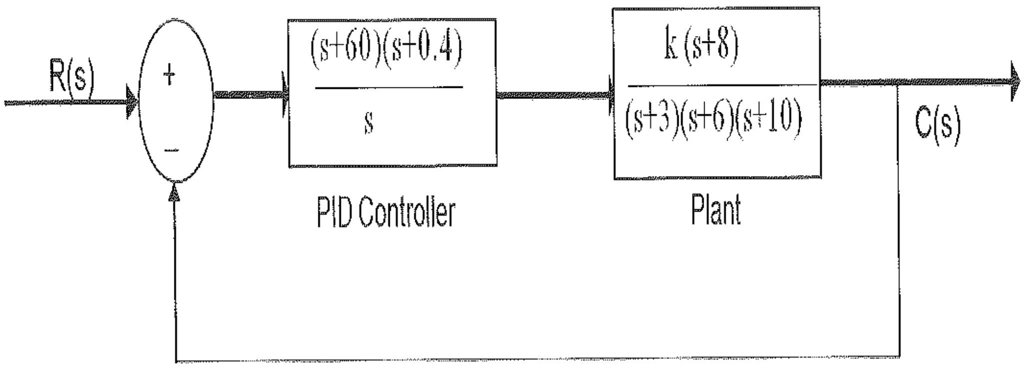

Figure 4 (a): Control system

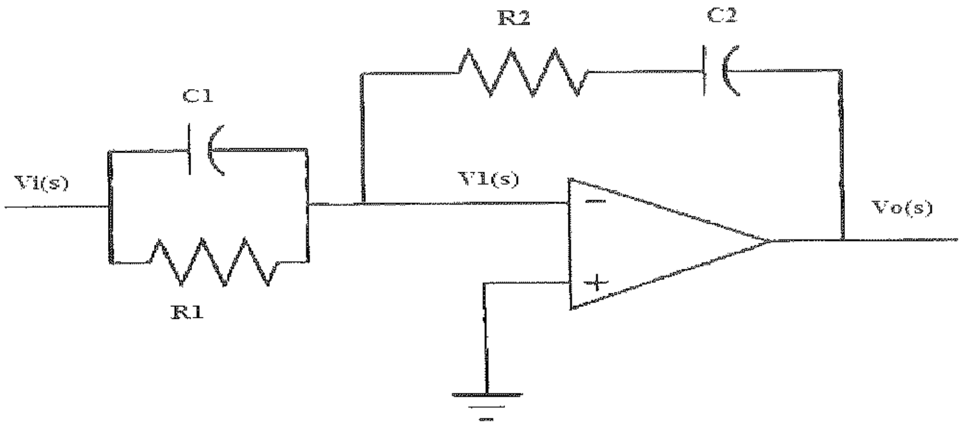

Figure 4 (b): PID Controller using operational amplifier

Please Turn Over

Select and answer only one question from questions 7 and question 8

Question 7

Figure 5 below shows an uncompensated unity feedback control system:

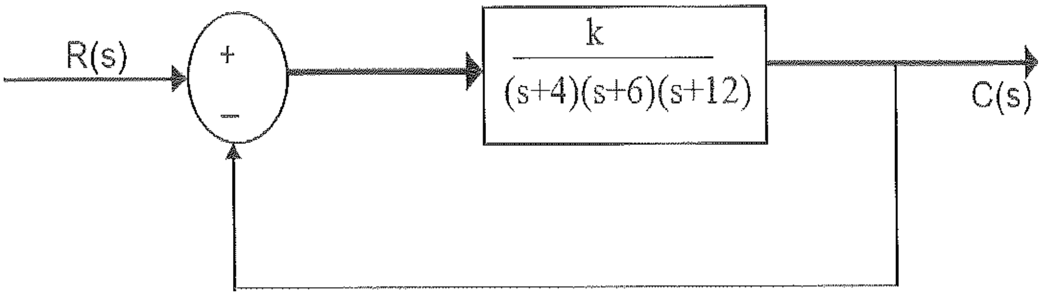

Figure 5: Uncompensated feedback control system

i) For the uncompensated feedback system of Figure 5 operating at 15% overshoot with step input, using root locus method compute the parameters listed in Table 1. ***(10 marks)***

Table 1: Parameters for uncompensated feedback system of Figure 5 at 15% overshoot

	Uncompensated system
Plant	$\frac{K}{(s+4)(s+6)(s+12)}$
Dominant poles	
K	
ζ	
w_n	
%OS	15
T_s	
T_p	
K_p	
$e(\infty)$	

ii) For the uncompensated feedback system of Figure 5, design a PD controller so that the system can operate at 15% overshoot with peak time that is two-third that of the uncompensated system. ***(10 marks)***

iii) For the uncompensated feedback system of Figure 5, design a PI controller so that the system can operate at 15% overshoot with zero steady-state error for a step input. ***(5 marks)***

Please Turn Over

Question 8

Figure 6 below shows an uncompensated unity feedback control system:

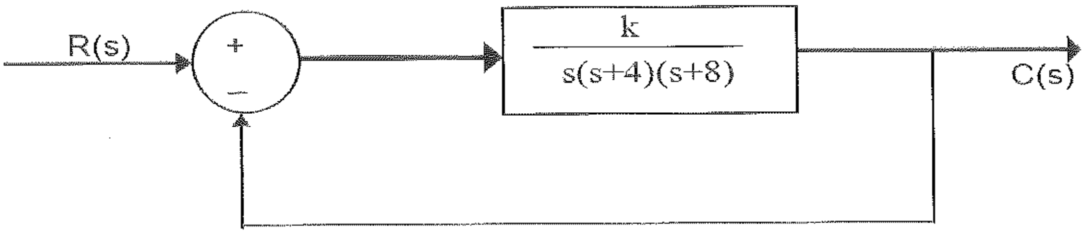

Figure 6: Uncompensated feedback control system

i) For the uncompensated feedback system of Figure 6 operating at 15% overshoot with ramp input, using root locus method compute the parameters listed in Table 2. ***(10 marks)***

Table 2: Parameters for uncompensated feedback system of Figure 6 at 20% overshoot

	Uncompensated system
Plant	$\frac{K}{s(s+4)(s+8)}$
Dominant poles	
K	
ζ	
w_n	
%OS	15
T_s	
T_p	
K_v	
$e(\infty)$	

ii) For the uncompensated feedback system of Figure 6, design a lead compensator so that the system can operate system at 15% overshoot with twofold reduction in settling time. ***(15 marks)***

THE END

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A: LAPLACE TRANSFORM OF COMMON FUNCTIONS

Time Functions f(t)	**Laplace Transform** $L\{f(t)\} = F(s)$
u(t), unit step. u(t) = 1	$1/s$
t, unit ramp	$1/s^2$
t^n	$n! / s^{n+1}$
e^{-at}	$1 / (s + a)$
$\cos \omega t$	$s / (s^2 + \omega^2)$
$\sin \omega t$	$\omega / (s^2 + \omega^2)$
$e^{-at} \cos \omega t$	$(s + a) / [(s + a)^2 + \omega^2]$
$e^{-at} \sin \omega t$	$\omega / [(s + a)^2 + \omega^2]$

B: LAPLACE TRANSFORM OPERATORS

Operation	**Time Domain**	**Laplace Domain**
Final Value theorem	$\lim_{t\to\infty} f(t)$	$\lim_{s\to 0} sF(s)$
Initial Value theorem	$\lim_{t\to 0} f(t)$	$\lim_{s\to\infty} sF(s)$
First Derivative	$\frac{d}{dt}f(t)$	$sF(s) - f(0)$
2nd Derivative	$\frac{d^2}{dt^2}f(t)$	$s^2F(s) - sf(0) - \frac{df(0)}{dt}$
nth Derivative	$\frac{d^n}{dt^n}f(t)$	$s^nF(s) - \sum_{r=1}^{n} \frac{d^{r-1}}{dt^{r-1}} f(0)\, s^{n-r}$
Complex Shift theorem	$e^{-at} f(t)$	$F(s + a)$
First Integral	$\int_0^t f(t)\, dt$	$(1/s)F(s)$
Multiplication by t	$t\, f(t)$	$-\frac{d}{ds}F(s)$
Division by t	$\frac{1}{t}f(t)$	$\int_s^\infty F(s)ds$

C: RELEVANT FORMULAE

System Performance & Specifications

1. $G(s) = \dfrac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$

2. $M_p = e^{-\left(\zeta\pi / \sqrt{1-\zeta^2}\right)}$

3. $\zeta = \sqrt{\dfrac{(\ln \frac{PO}{100})^2}{\pi^2 + (\ln \frac{PO}{100})^2}}$

4. $t_p = \dfrac{\pi}{\omega_n \sqrt{1 - \zeta^2}} = \dfrac{\pi}{\omega_d}$

5. $T_s = \dfrac{4}{\zeta\omega_n}$

6. $\Phi_M = 90 - \tan^{-1}\dfrac{\sqrt{-2\zeta^2 + \sqrt{1+4\zeta^4}}}{2\zeta}$

   $= \tan^{-1}\dfrac{2\zeta}{\sqrt{-2\zeta^2 + \sqrt{1+4\zeta^4}}}$

D: Z -TRANSFORM

Function No	f(t) $t \geq 0$ f(t) = 0 for $t < 0$	Laplace Transform F(s) = L{f(t)}	f[k], f[kT] $k \geq 0$ f[kT] = 0 for $k < 0$	Z-Transform F(z) = Z{f[kT]}
1	Impulse function $\delta(t)$	1	$\delta[k]$	1
2	Unit step function u(t)	$\frac{1}{s}$	u[k]	$\frac{z}{z-1} = \frac{1}{1-z^{-1}}$
3	-		a^k	$\frac{z}{z-a} = \frac{1}{1-az^{-1}}$
4	-		ka^k	$\frac{az}{(z-a)^2} = \frac{az^{-1}}{(1-az^{-1})^2}$
5	t	$\frac{1}{s^2}$	kT	$\frac{Tz}{(z-1)^2} = \frac{Tz^{-1}}{(1-z^{-1})^2}$
6	t^2	$\frac{2}{s^3}$	$(kT)^2$	$\frac{T^2 z(z+1)}{(z-1)^3}$
7	e^{-at}	$\frac{1}{(s+a)}$	e^{-akT}	$\frac{z}{z-e^{-aT}} = \frac{1}{1-z^{-1}e^{-aT}}$
8	Sin(ωt)	$\frac{\omega}{(s^2+\omega^2)}$	Sin[ωkT]	$\frac{z\sin(\omega T)}{z^2-2z\cos(\omega T)+1}$
9	Cos(ωt)	$\frac{s}{(s^2+\omega^2)}$	Cos[ωkT]	$\frac{z[z-\cos(\omega T)]}{z^2-2z\cos(\omega T)+1}$
10	te^{-at}	$\frac{1}{(s+a)^2}$	kTe^{-akT}	$\frac{Tze^{-aT}}{(z-e^{-aT})^2} = \frac{Tz^{-1}e^{-aT}}{(1-z^{-1}e^{-aT})^2}$
11	e^{-at} Sin(ωt)	$\frac{\omega}{(s+a)^2+\omega^2}$	e^{-akT} Sin[ωkT]	$\frac{ze^{-aT}\sin(\omega T)}{z^2-2ze^{-aT}\cos(\omega T)+e^{-2aT}}$
12	e^{-at} Cos(ωt)	$\frac{(s+a)}{(s+a)^2+\omega^2}$	e^{-akT} Cos[ωkT]	$\frac{z^2-ze^{-aT}\cos(\omega T)}{z^2-2ze^{-aT}\cos(\omega T)+e^{-2aT}}$

E: ASYMPTOTIC PLOTS ERRORS

Asymptotic Plot Errors for $(1 + j\omega_i\tau_{zi})$

Corner Freq. $\omega_c = \frac{1}{\tau_z}$	$\frac{\omega_c}{5}$	$\frac{\omega_c}{2}$	ω	2ω	5ω
Mag. error dB	0.17	0.96	3.0	0.96	0.17
Phase angle error	11.3°	0.8°	0.0°	-0.8°	-11.3°

Asymptotic Plot Errors for $\frac{1}{1 + j\omega_i\tau_{pi}}$

Corner Freq. $\omega_c = \frac{1}{\tau_z}$	$\frac{\omega_c}{5}$	$\frac{\omega_c}{2}$	ω	2ω	5ω
Mag. error dB	-0.17	-0.96	-3.0	-0.96	-0.17
Phase angle error	-11.3°	-0.8°	0.0°	0.8°	11.3°

EEE743 Answer Booklet for Question 4(a)