FIJI NATIONAL UNIVERSITY

College of Engineering, Science and Technology

School of Electrical & Electronics Engineering

EEE 694 – Engineering Mathematics III

Semester I, 2017

FINAL EXAMINATION

Programme: Bachelor of Engineering, 2nd year

Time Allowed: 3 HOURS 10 MINS

Time: **Date:**

Instructions to Students:

1. The question paper contains three (3) pages.
2. There are thirteen (13) questions in the paper. Answer any ten (10) questions.
3. You are allowed 10 minutes extra reading time during which you are NOT allowed to write.
4. This exam is worth 50% of your overall marks.
5. Answer questions neatly on a new page and clearly number the question attempted.
6. Students may use a calculator, provided it is silent and non-programmable.
7. If you use extra sheets of paper, attach it securely to the answer booklet.

[1] Find the charge and the current for $t > 0$ in a series RC circuit where $R = 10\Omega$, $C = 4 \times 10^{-3}F$ and $E = 85cos150t\,V$. Assume that when the switch is closed at $t = 0$, the charge on the capacitor is $-0.005C$.

[10]

[2] (a) A series of RL circuit with $R = 50\Omega$ and L=10H has a constant voltage $V = 100V$ applied at $t = 0$ by the closing of a switch. Find the equation for current i(t).

(b) Find the standard matrix for the linear operator T , where

$$T(x_1, x_2, x_3) = (x_1 + 2x_2 + x_3, x_1 + 5x_2, x_3).$$

[5+5]

[3] (a) Solve the linear ordinary differential equation: $xy' = 2y + e^x x^3$.

(b) Let the electric equipotential lines between two concentric cylinders with the z-axis in space be given by $u(x, y) = x^2 + y^2 = c$. Find their orthogonal trajectories (the curves of electric force).

[5+5]

[4] (a) Solve the first order ordinary differential equation $(x^2 + y^2)dx - 2xydy = 0$.

(b) Test whether an exact differetial equation. If so, solve

$$\cos(x + y)\, dx + (3y^2 + 2y + \cos(x + y)\, dy = 0.$$

[5+5]

[5] (a) Solve the initial value problem $y'' + y' - 2y = 0, y(0) = 4, y'(0) = -5$.

(b) Verify and explain why $y = e^{-2x}$ is a solution of $y'' - y' - 6y = 0$ but xe^{-2x} is not.

[5+5]

[6] (a) Find a real general solution of Euler –Cauchy equation $(x^2D^2 - 3xD + 10I)y = 0$.

(b) Solve the initial value problem $y'' + 3y' + 2.25y = -10e^{-1.5\,x}$, $y(0) = 1, y'(0) = 0$.

[5+5]

[7] (a) Solve the differential equation $y'' + y = sex$ by method of variation of parameters.

(b) Solve the ordinary differential euqation $y''' - 9y'' + 27y' - 27y = 27sin3x$.

[5+5]

[8] Find the eigenvalues, eigen space and eigen vectors of $A = \begin{pmatrix} 13 & 5 & 2 \\ 2 & 7 & -8 \\ 5 & 4 & 7 \end{pmatrix}$.

[10]

[9] Determine whether matrix $A = \begin{pmatrix} -1 & 4 & -2 \\ -3 & 4 & 0 \\ -3 & 1 & 3 \end{pmatrix}$ is diagonalizable. If so, find a matrix P that diagonalizes A, and determine $P^{-1}AP$..

[10]

[10] Find the current I(t) in an LC-circuit when L = 0.5H , C = 0.005 F , which is connected to a source of EMF E(t) = sint Volts . Assuming zero initial current and charge.

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[10]

[11] Solve the system

$$y_1' = 4y_1 + y_3$$
$$y_2' = -2y_1 + y_2$$
$$y_3' = -2y_1 + y_3.$$

Find the solution that satisfies the initial conditions $y_1(0) = -1, y_2(0) = 1, y_3(0) = 0$.

[10]

[12] Let $T: R^3 \to R^3$ be the linear operator defined by the formula,

$$T(x_1, x_2, x_3) = (3x_1 + x_2, -2x_1 - 4x_2 + 3x_3, 5x_1 + 4x_2 - 2x_3).$$

Determine whether T is one-to-one; if so, find $T^{-1}(x_1, x_2, x_3)$.

[10]

[13] The linear transformation $T: R^4 \to R^4$ is defined by

$T(x_1, x_2, x_3, x_4) = (5x_1 - 2x_2 + x_3, -2x_1 - 4x_3 + x_4, x_1 - 4x_2 - 11x_3 + 2x_4, x_2 + 2x_3)$.

Find rank of T and nullity of T.

[10]

--:THE END:--