FIJI NATIONAL UNIVERSITY

College of Engineering, Science and Technology

School of Electrical and Electronics Engineering

Advanced Diploma in Engineering (Electrical & Electronics)

EEE608 Engineering Computations 2

Final Exam Question Paper

Semester 2, 2016

Time Allocated: **3 hours**
(with extra 10 minutes for Reading Time)
Total Marks: 100

Instructions

1. This paper has four sections and is 5 pages long, including this cover page. **All sections are compulsory.** The marks allocation is given below.

Section A	20 marks
Section B	30 marks
Section C	30 marks
Section D	20 marks

2. **Answer all questions in the Answer Booklet provided and show all workings where necessary. This exam contributes 50% to our overall mark.**
3. Begin each section on a fresh page and use both sides of the sheet.
4. Write your candidate I.D. Number at the top of each attached sheet.
5. Non-programmable calculators may be used.

Section A Vector Differential Calculus (20 marks)

1. Find the length of the curve.

$$\mathbf{r}(t) = \langle 2t, t^2, \tfrac{1}{3}t^3 \rangle, \qquad 0 \le t \le 1\,.$$

(5 marks)

2. According to the **Ideal Gas Law**, the pressure, temperature, and volume of a confined gas are related by $P = kT/V$, where k is a constant. Use differentials

$$df = f_x\,dx + f_y\,dy$$

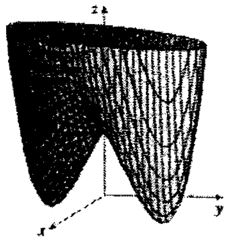
to approximate the percentage change in pressure if the temperature of a gas is increased 3% and the volume is increased 5%.

(5 marks)

3. Find the local maximum and minimum values and saddle point of

$$f(x, y) = x^4 + y^4 - 4xy + 1.$$

[Hint: $D = f_{xx}f_{yy} - (f_{xy})^2$]

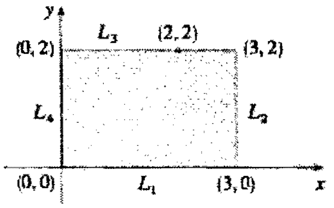


(5 marks)

4. Find the absolute maximum and minimum values of the function

$$f(x, y) = x^2 - 2xy + 2y$$

on the rectangle $D = \{(x, y) \mid 0 \le x \le 3, 0 \le y \le 2\}$.
[Hint: Find the critical points in the rectangle and the absolute max/min for the paths L_1, L_2, L_3 and L_4 and compare.]



(5 marks)

Section B Multiple Integrals (30 marks)

1. Given that $\mathbf{H} = x^2\mathbf{i} + y^2\mathbf{j}$, evaluate

$$\int_L \mathbf{H} \cdot d\mathbf{l},$$

where L is along the curve $y = x^2$ from $(0,0)$ to $(1,1)$.
[Hint: $d\mathbf{l} = dx\,\mathbf{i} + dy\,\mathbf{j} + dz\,\mathbf{k}$.]

(5 marks)

2. Calculate the area of the following surface using the differential surface area $d\mathbf{S}$.
[Hint: $d\mathbf{S} = r\,dz\,d\theta\,\mathbf{a}_r + dz\,dr\,\mathbf{a}_\theta + r\,dr\,d\theta\,\mathbf{a}_z$]

$$r = 2, \qquad 0 < z < 5, \qquad \pi/3 < \theta < \pi/2\,.$$

(5 marks)

3. Use triple integration in cylindrical coordinates to find the volume of the solid G that is bounded above by the hemisphere

$$z = \sqrt{25 - x^2 - y^2},$$

below by the xy-plane, and laterally by the cylinder $x^2 + y^2 = 9$.
[Hint: $dV = r\,dz\,dr\,d\theta$]

(10 marks)

4. Convert to spherical coordinates and evaluate

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{1}{1 + x^2 + y^2 + z^2}\,dz\,dy\,dx.$$

[Hint: $x = \rho \sin\phi \cos\theta$, $y = \rho \sin\phi \sin\theta$, $z = \rho\cos\phi$]

(10 marks)

Section C Vector Integral Calculus (30 marks)

1. Use **Green's Theorem** to evaluate the integral

$$\oint_C \left(x^2 - y^2\right) \, dx + x \, dy,$$

where C is the circle $x^2 + y^2 = 9$.

(10 marks)

2. Apply **Divergence Theorem**

$$\oiint_S \mathbf{F} \cdot d\mathbf{S} = \iiint_V (\nabla \cdot \mathbf{F}) \, dV$$

to find the net flux passing through $\mathbf{F} = xy^2\mathbf{i} + y^3\mathbf{j} + y^2 z\mathbf{k}$ and S is the surface of the cuboid defined by $0 < x < 1$, $0 < y < 1$, $0 < z < 1$.
[Hint: $\nabla \cdot \mathbf{F} = \frac{\partial}{\partial x}(F_x) + \frac{\partial}{\partial y}(F_y) + \frac{\partial}{\partial z}(F_z)$]

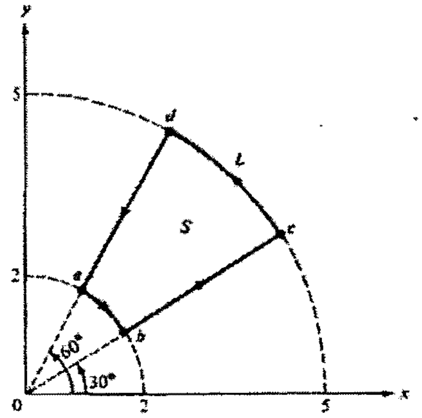
(10 marks)

3. If $\mathbf{F} = r\cos\theta \mathbf{a}_r + \sin\theta \mathbf{a}_\theta$, utilize **Stokes's Theorem**

$$\oint \mathbf{F} \cdot d\mathbf{l} = \iint_S (\nabla \times \mathbf{F}) \cdot d\mathbf{S}$$

to evaluate $\oint \mathbf{F} \cdot d\mathbf{l}$ around the path L shown below. Note that in cylindrical coordinates,

$$\nabla \times \mathbf{F} = \frac{1}{r} \begin{vmatrix} \mathbf{a}_r & r\mathbf{a}_\theta & \mathbf{a}_z \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial z} \\ F_r & rF_\theta & F_z \end{vmatrix}$$



(10 marks)

Section D Laplace Transform and Fourier Series (20 marks)

1. Find the Laplace Transform of

$$f(t) = \cos \omega t$$

using the result of Transform of Second Derivatives

$$\mathscr{L}\{f''\} = s^2\mathscr{L}\{f\} - sf(0) - f'(0).$$

(5 marks)

2. Use Laplace transform to solve the following Initial Value Problem.

$$y'' - 5y' + 4y = 0, \qquad y(0) = 0, \qquad y'(0) = 1.$$

(7 marks)

3. Determine if the given functions are odd or even or neither. Find the Fourier series for the function. Show details of your work.

$$f(x) = \begin{cases} 1, & -2 < x < 0 \\ -1, & 0 < x < 2 \end{cases}$$

with period $p = 2L$.

(8 marks)

The End
(All the best for the paper)