FIJI NATIONAL UNIVERSITY

College of Engineering, Science and Technology (CEST)
School of Electrical & Electronic Engineering

EEE529 CONTROL SYSTEMS

Final Examination

Friday 11th November, 2016 1400 - 1710 hours Venue: C106

INSTRUCTIONS TO CANDIDATES

1. Candidates are reminded that they should have no books, notes, paper or other material in their possession unless their use is specifically permitted by "Instructions to Candidates" set out below.
2. Reading time is of 10 minutes duration.
3. Examination time is of 3 hours duration.
4. This paper consists of 8 questions printed on 10 pages.
5. Attempt all 8 questions. Each question may carry a different mark.
6. A set of Laplace Transforms Table is attached.
7. The datasheet for the 74LS153 Multiplexer is at the back of this paper..
8. Write your candidate number at the top of each attached sheet.
9. Start each question on a new page.
10. Non-Programmable Calculators may be used.
11. Mobile phones are not allowed inside the examination venue.

FORMULA SHEET

1. Integrating Factor $R = e^{\int P(t)dt}$; $Rf(t) = \int RQ(t)dt$
2. $\int udv = uv - \int vdu$
3. $G(s) = \dfrac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$
4. $\%OS = e^{-\left(\frac{\xi\pi}{\sqrt{1-\xi^2}}\right)} \times 100\%$
5. $T_p = \dfrac{\pi}{\omega_n\sqrt{1-\xi^2}}$
6. $T_s = \dfrac{4}{\xi\omega_n}$
7. $s = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
8. $e_n = \sqrt{4kT(BW)R}$
9. $P_n = kT(BW)$
10. $CLTF = \dfrac{G(s)}{1 + G(s)H(s)}$
11. $f(t) = a_0 + \sum_{n=1}^{\infty}\left(a_n \cos\frac{n\pi}{L}t + b_n \sin\frac{n\pi}{L}t\right)$; $a_0 = \frac{1}{T}\int_0^T f(t)dt = \frac{1}{2L}\int_0^{2L} f(t)dt$;

    $a_n = \frac{2}{T}\int_0^T f(t)\cos n\omega t\, dt = \frac{1}{L}\int_0^{2L} f(t)\cos\frac{n\pi}{L}t\, dt$;

    $b_n = \frac{2}{T}\int_0^T f(t)\sin n\omega t\, dt = \frac{1}{L}\int_0^{2L} f(t)\sin\frac{n\pi}{L}t\, dt$
12. $C = 2B\log_2 M$; $C = B\log_2\left(1 + \frac{S}{N}\right)$
13. $q = CV$; $i(t) = \frac{v(t)}{R}$; $i(t) = \frac{dq(t)}{dt}$; $i(t) = C\frac{dv(t)}{dt}$; $i(t) = \frac{1}{L}\int v(t)\, dt$; $w_C = \frac{1}{2}CV^2$
14. $v(t) = L\frac{di(t)}{dt}$; $w_L = \frac{1}{2}LI^2$

QUESTION 1: LAPLACE TRANSFORMS; UNIT STEP FUNCTION [TOTAL: 12 MARKS]

(a) Determine the s-domain equivalents of the following functions using the given table of Laplace Transform.

(i) $f(t) = 5e^{-3t} + 3t^5 - 2\cosh 3t$ **[2 marks]**

(ii) $f(t) = 7e^{3t}\cos 9t$ **[2 marks]**

(iii) $f(t) = \begin{cases} 0, & t < 8 \\ (t-8)^4 & t > 8 \end{cases} = u(t-8)(t-8)^4$ **[2 marks]**

(b) Sketch the functions,

(i) $f(t) = 7u(t) + u(t-10)$ **[2marks]**

(ii) $f(t) = 12u\left(t - \frac{\pi}{2}\right)\sin t, \quad 0 \le t \le 2\pi$ **[2marks]**

(iii) $f(t) = u(t).t^2, \quad f(t+2) = f(t); \quad 0 \le t \le 6$ **[2marks]**

QUESTION 2: STEP RESPONSE OF FIRST ORDER *RC* NETWORK [TOTAL: 13 MARKS]

(a) The 1st Order *RC* circuit in
(b) Figure 1 has zero initial conditions. The switch S1 is closed at time $t = 0$.

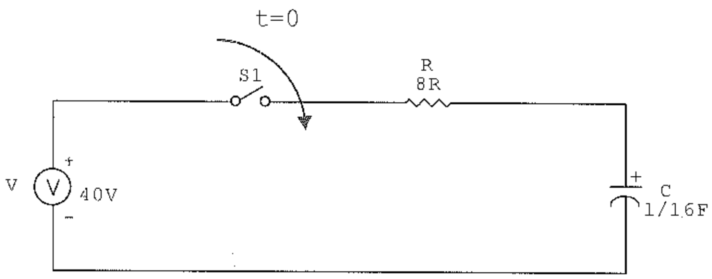


Figure 1: 1st Order *RC* network

(i) Determine the solution, $q(t)$, by applying Kirchoff's Voltage Law (KVL) and using conventional Calculus. Identify the Steady State and the Transient State. **[4 marks]**

(ii) Resolve for $q(t)$ using Laplace Transforms. **[4 marks]**

(iii) Derive the expression for the instantaneous current, $i(t)$. **[2 marks]**

(iv) Given the step response of a 1st Order System, $C(s) = \dfrac{5}{s(s+2)}$, use the method of Poles & Zeros to determine time-domain response. **[3 marks]**

QUESTION 3: SECOND ORDER SYSTEMS **[TOTAL: 12 MARKS]**

(a) Given an Underdamped system, find the following if the Transfer Function is

$$G(s) = \frac{36}{s^2 + 9s + 36}$$

(i) Peak time (T_p) **[3 marks]**

(ii) Percentage Overshoot **[2 marks]**

(iii) Settling time (T_s) **[2 marks]**

(b) Refer to the Series *RLC* circuit shown in Figure 2. The output is taken across the capacitor, *C*. Use Laplace Transforms to derive the solution of q(t). Assume zero initial conditions. **[5 marks]**

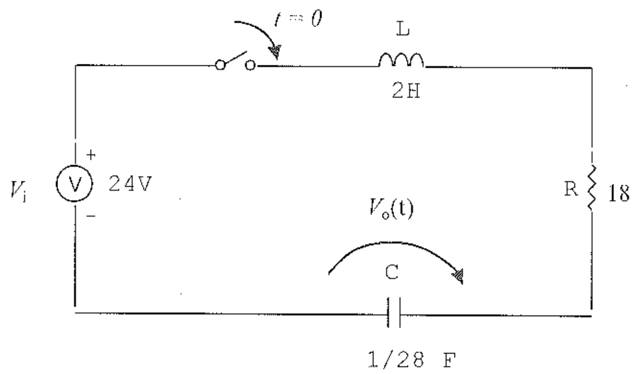


Figure 2: Series *RLC* circuit

QUESTION 4 : ROOT LOCUS [TOTAL: 10 MARKS]

Consider the system shown in
Figure 3.

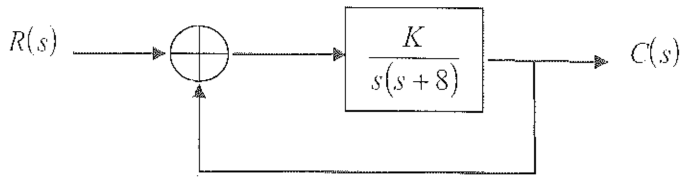


Figure 3: A 2nd Order System

(a) Derive the Characteristic Equation in quadratic form. **[3 marks]**

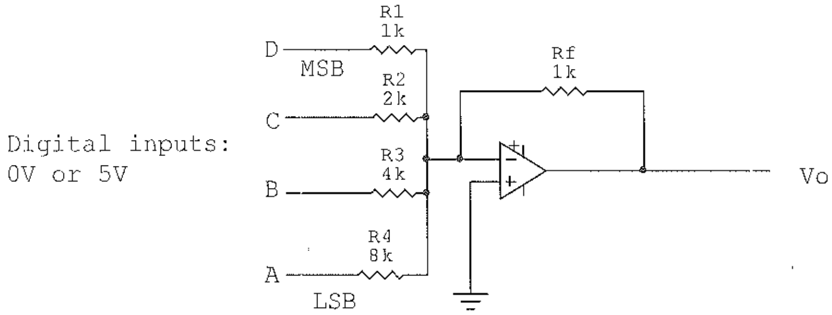
(b) Sketch the Root Locus. [Hint: Obtain roots for k = 0, 4, 8, 12, 16, 20] **[5 marks]**

(c) Find the value(s) of *k* that make the system overdamped, but will still keep the system stable. **[2 marks]**

QUESTION 5 : SIGNAL CONDITIONING [TOTAL: 13 MARKS]

(a) Consider a 4-bit DAC using an op-amp summing amplifier with binary-weighted resistors as shown in

(b) Figure 1. Assume that **Logic 1 = 10V** whilst Logic 0 = 0V.



[5 marks]

Figure 4: 4-bit DAC

Determine the step size (resolution) and find the output voltages for all the combinations of the input code, from DCBA = 0000 to 1111. Tabulate your results as shown in Table 1:

D	C	B	A		V_{out} (v)
0	0	0	0		
0	0	0	1		
0	0	1	0		
0	0	1	1		
0	1	0	0		
0	1	0	1		
0	1	1	0		
0	1	1	1		
1	0	0	0		
1	0	0	1		
1	0	1	0		
1	0	1	1		
1	1	0	0		
1	1	0	1		
1	1	1	0		
1	1	1	1		

Table 1: DAC Table

(c) Describe concisely how the analogue voltage, V_A = 7V, is converted to its digital equivalent by the ADC illustrated in Figure 5. [4 marks]

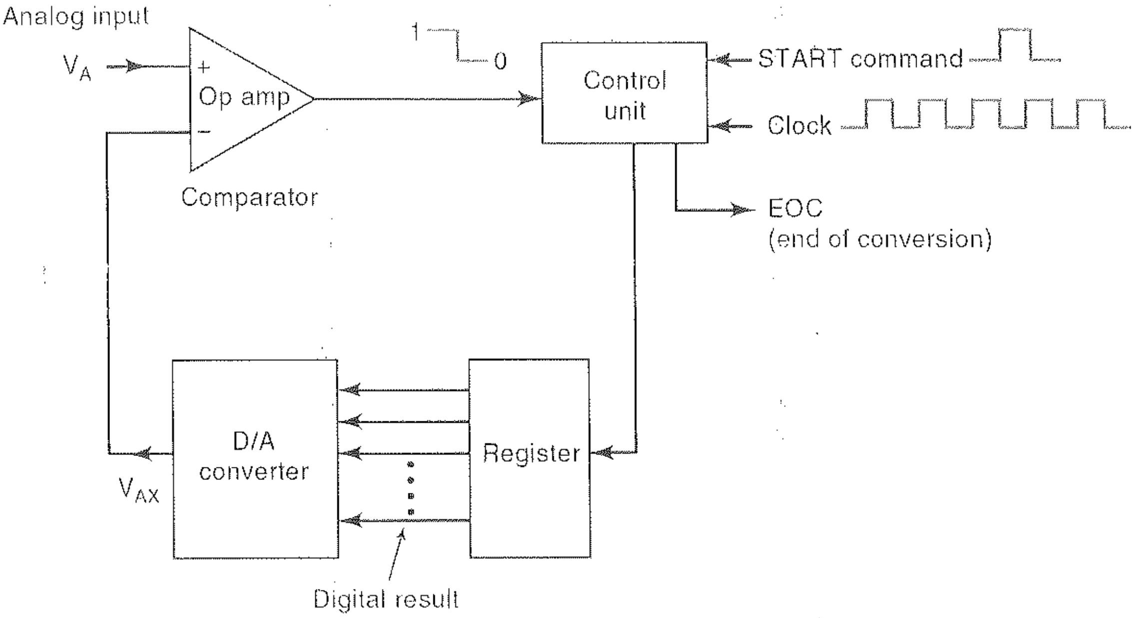
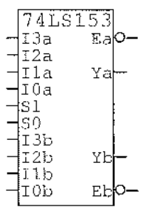


Figure 5: ADC

(d) Refer to Table 1, showing the Function Table for the Full Adder (Binary Adder). Design a circuit using the 74LS153 Multiplexer to implement the Sum (S) and the Carry Out (C_{out}). Both outputs S and C_{out} are to be present at any time. (Note: Datasheet for 74LS153 is provided).

Table 2: Binary Adder

A	B	K	C_{out}	S
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1



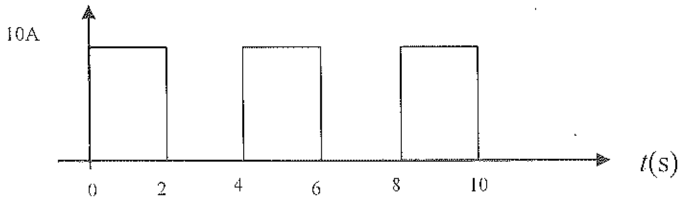
[4 marks]

QUESTION 6 [FOURIER SERIES, NOISE, BANDWIDTH] [TOTAL: 22 MARKS]

(a) Refer to the current pulse train shown.

(i) Use the Fourier Series to synthesize the equivalent of the function *i(t)*, shown in Figure 7. Determine the first 5 terms. The pulse train represents the function,

$$i(t) = \begin{cases} 10, & 0 \le t < 2 \\ 0, & 2 \le t < 4 \end{cases}; \quad i(t+4) = i(t)$$



[6 marks]

Figure 6: Current pulse train

(ii) Predict the amplitude and the frequency for the term when n = 11. **[2 marks]**

(b) A Karaoke system has a bandwidth of 1 MHz, voltage gain of 150 and an input resistance of 5 kΩ. The room temperature is 27 °C. The singers produce an input audio signal of 4 uV_{rms}. With Boltzmann's constant at 1.38×10^{-23} J/K, determine the following

(i) White [Johnson] Noise Power **[3 marks]**

(ii) RMS input noise level **[2 marks]**

(iii) Audio output level **[2 marks]**

(iv) RMS output noise level **[2 marks]**

(b) The bandwidth, *B*, of an Audio Recording studio is 20 kHz.

(i) Calculate the Maximum Data Transfer Rate (Capacity) using the Nyquist (Hartley) equation. There are 8 signalling levels and assume a noiseless environment. **[2 marks]**

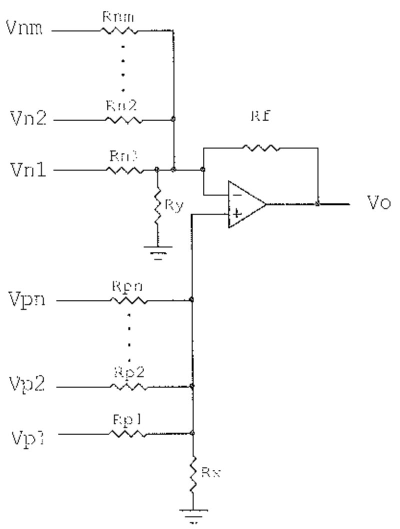
(ii) If the S/N ratio is 50 dB, determine the maximum information rate, *C*, predicted by Shannon's law, $C = B\log_2\left(1+\frac{S}{N}\right)$.

[3 marks]

QUESTION 7 OPERATIONAL AMPLIFIERS – ANALOGUE COMPUTERS [TOTAL: 10 MARKS]

Design an Analogue Computer using Operational amplifiers to solve the second order differential equation, $m'' = -6m' + 3m - 4\sin 2t$. Use integrators whose time constant $RC = 1$. Assume the initial conditions m'(0) = 1 and m(0) = 0.5 V. Provide a block diagrammatic representation of the circuit first. State any assumptions you make. Do note the General Add-Subtract circuit shown in Figure 7, and the parameters that need to be evaluated.

[10 marks]



$$V_o = \sum_{i=1}^{n} A_i v_{pi} - \sum_{i=1}^{m} B_i v_{ni}$$

where, $A_i = \frac{R_f}{R_{pi}}, B_i = \frac{R_f}{R_{ni}}$

Let $A = \sum A_i, B = \sum B_i$

Let $C = A - B - 1$

If

$$C > 0 \quad R_x = \infty \qquad R_y = \frac{R_f}{C}$$

$$C < 0 \quad R_x = -\frac{R_f}{C} \quad R_y = \infty$$

Figure 7: General Add-Subtract circuit

QUESTION 8: PARALLEL *RLC* NETWORK [TOTAL: 12 MARKS]

(a) Analyse the Parallel *RLC* network shown.

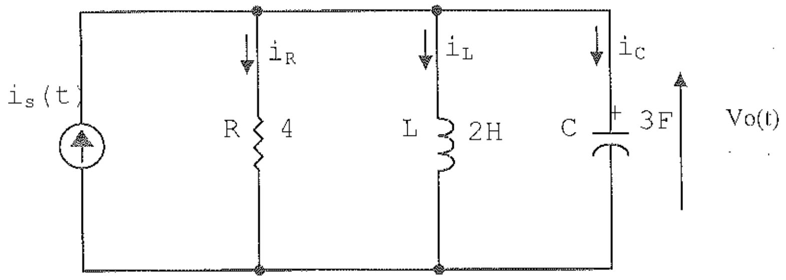


Figure 8: Parallel *RLC* network

(i) Derive the mathematical model in the s-domain. Assume zero initial conditions. [4 marks]

(ii) Construct the block diagram of the network, then reduce it to its simplest Closed-loop form using the block reduction technique. **[4 marks]**

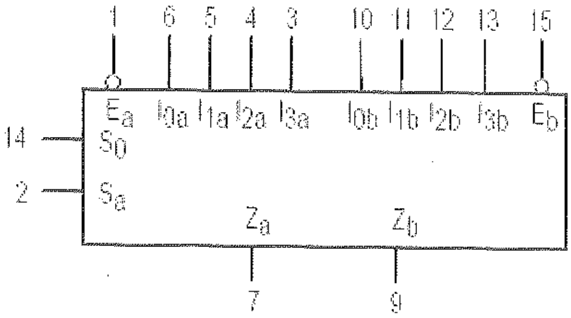
(iii) Find the Closed-loop Transfer Function. Represent this via a block diagram. **[4 marks]**

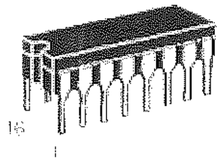
[THE END]

SELECT INPUTS		INPUTS (a or b)					OUTPUT
S_0	S_1	$\overline{E}$	I_0	I_1	I_2	I_3	Z
X	X	H	X	X	X	X	L
[illegible]	L	L	L	X	X	X	L
[illegible]	L	L	H	X	X	X	H
H	L	L	X	L	X	X	L
H	L	L	X	H	X	X	H
L	H	L	X	X	L	X	L
L	H	L	X	X	H	X	H
H	H	L	X	X	X	L	L
H	H	L	X	X	X	H	H

H = HIGH Voltage Level
L = LOW Voltage Level
X = Don't Care

TRUTH TABLE





V_{CC} = PIN 16
GND = PIN 8

74LS153

TABLE OF LAPLACE TRANSFORMS

	$F(s)=L[f(t)]$	$f(t),\quad t \ge 0$
1.	1	$\delta(t_0)$, Unit Impulse at $t = t_0$
2.	$\frac{1}{s}$	1, Unit Step
3.	$\frac{n!}{s^{n+1}}$	t^n
4.	$\frac{1}{s+a}$	e^{-at}
5.	$\frac{1}{(s+a)^n}$	$\frac{1}{(n-1)!}t^{n-1}e^{-at}$
6.	$\frac{a}{s(s+a)}$	$1-e^{-at}$
7.	$\frac{1}{(s+a)(s+b)}$	$\frac{1}{(b-a)}\left(e^{-at}-e^{-bt}\right)$
8.	$\frac{s+\alpha}{(s+a)(s+b)}$	$\frac{1}{(b-a)}\left[(\alpha-a)e^{-at}-(\alpha-b)e^{-bt}\right]$
9.	$\frac{ab}{s(s+a)(s+b)}$	$1-\frac{b}{(b-a)}e^{-at}+\frac{a}{(b-a)}e^{-bt}$
10.	$\frac{1}{(s+a)(s+b)(s+c)}$	$\frac{e^{-at}}{(b-a)(c-a)}+\frac{e^{-bt}}{(c-a)(a-b)}+\frac{e^{-ct}}{(a-c)(b-c)}$
11.	$\frac{s+\alpha}{(s+a)(s+b)(s+c)}$	$\frac{(\alpha-a)e^{-at}}{(b-a)(c-a)}+\frac{(\alpha-b)e^{-bt}}{(c-b)(a-b)}+\frac{(\alpha-c)e^{-ct}}{(a-c)(b-c)}$
12.	$\frac{ab(s+\alpha)}{s(s+a)(s+b)}$	$\alpha-\frac{b(\alpha-a)}{(b-a)}e^{-at}+\frac{a(\alpha-a)}{(b-a)}e^{-bt}$
13.	$\frac{\omega}{s^2+\omega^2}$	$\sin\omega t$
14.	$\frac{s}{s^2+\omega^2}$	$\cos\omega t$
15.	$\frac{s+a}{s^2+\omega^2}$	$\frac{\sqrt{\alpha^2+\omega^2}}{\omega}\sin(\omega t+\phi)$, $\phi=\tan^{-1}\left(\frac{\omega}{\alpha}\right)$
16.	$\frac{\omega}{(s+a)^2+\omega^2}$	$e^{-at}\sin\omega t$

Please turn over...

$F(s) = L[f(t)]$	$f(t), \quad t \geq 0$
17. $\dfrac{(s+a)}{(s+a)^2+\omega^2}$	$e^{-at}\cos\omega t$
18. $\dfrac{s+\alpha}{(s+a)^2+\omega^2}$	$\dfrac{1}{\omega}\left[(\alpha-a)^2+\omega^2\right]^{\frac{1}{2}} e^{-at}\sin(\omega t+\phi)$, $\phi=\tan^{-1}\left(\dfrac{\omega}{\alpha-a}\right)$
19. $\dfrac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$	$\dfrac{\omega_n}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin\omega_n\sqrt{1-\zeta^2}\,t,\ \zeta<1$
20. $\dfrac{1}{s\left[(s+a)^2+\omega^2\right]}$	$\dfrac{1}{a^2+\omega^2}+\dfrac{1}{\omega\sqrt{a^2+\omega^2}}e^{-at}\sin(\omega t-\phi)$, $\phi=\tan^{-1}\left(\dfrac{\omega}{-a}\right)$
21. $\dfrac{\omega_n^2}{s\left(s^2+2\zeta\omega_n s+\omega_n^2\right)}$	$1-\dfrac{1}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin\left(\omega_n\sqrt{1-\zeta^2}\,t+\phi\right)$, $\phi=\cos^{-1}\zeta,\zeta<1$
22. $\dfrac{(s+\alpha)}{s\left[(s+a)^2+\omega^2\right]}$	$\dfrac{\alpha}{a^2+\omega^2}+\dfrac{1}{\omega}\left[\dfrac{(\alpha-a)^2+\omega^2}{a^2+\omega^2}\right]^{\frac{1}{2}}e^{-at}\sin(\omega t+\phi)$, $\phi=\tan^{-1}\left(\dfrac{\omega}{\alpha-a}\right)-\tan^{-1}\left(\dfrac{\omega}{-a}\right)$
23. $\dfrac{1}{(s+c)\left[(s+a)^2+\omega^2\right]}$	$\dfrac{e^{-ct}}{(c-a)^2+\omega^2}+\dfrac{e^{-at}\sin(\omega t+\phi)}{\omega\left[(c-a)^2+\omega^2\right]^{\frac{1}{2}}}$, $\phi=\tan^{-1}\left(\dfrac{\omega}{c-a}\right)$

24. $\dfrac{a}{s^2 - a^2}$	$\sinh at$
25. $\dfrac{s}{s^2 - a^2}$	$\cosh at$
26. $\dfrac{\omega^2}{s\left(s^2 + \omega^2\right)}$	$1 - \cos \omega t$
27. $\dfrac{\omega^3}{s^2\left(s^2 + \omega^2\right)}$	$\omega t - \sin \omega t$
28. $\dfrac{2\omega^3}{\left(s^2 + \omega^2\right)^2}$	$\sin \omega t - \omega t \cos \omega t$

$F(s) = L[f(t)]$		$f(t), \quad t \geq 0$
29. $\dfrac{s}{(s^2+\omega^2)^2}$		$\dfrac{t}{2\omega}\sin\omega t$
30. $\dfrac{s^2}{(s^2+\omega^2)^2}$		$\dfrac{1}{2\omega}(\sin\omega t + \omega t\cos\omega t)$
31. $\dfrac{s}{(s^2+a^2)(s^2+b^2)}$	$(a^2 \neq b^2)$	$\dfrac{1}{b^2-a^2}(\cos at - \cos bt)$

LAPLACE TRANSFORM OF DERIVATIVES

32. $sF(s) - f(0)$	$f'(t)$, First Derivative
33. $s^2F(s) - sf(0) - f'(0)$	$f''(t)$, Second Derivative

GENERAL PROPERTIES OF LAPLACE TRANSFORMS

1. Linearity $\quad L[af_1(t) + bf_2(t)] = aF_1(s) + bF_2(s)$

2. Scaling $\quad L[f(at)] = \dfrac{1}{a}F\left(\dfrac{s}{a}\right)$

3. Transform of a Derivative

$$L\left[f^{(n)}(t)\right] = s^nF(s) - s^{n-1}f(0) - s^{n-2}f'(0) - \ldots - sf^{(n-2)}(0) - f^{(n-1)}(0)$$

4. First Shifting Theorem $\quad L\left[e^{at}f(t)\right] = F(s-a)$
   (Complex Translation; s-shifting)

5. Second Shifting Theorem $\quad L[u(t-a)f(t-a)] = e^{-as}F(s)$
   (Real Translation; t—shifting)

6. Integration $\quad L\left[\int_0^t f(\tau)d\tau\right] = \dfrac{1}{s}F(s)$

7. Complex Differentiation $\quad L[tf(t)] = -\dfrac{dF(s)}{ds}$

8. Final Value Theorem $\quad f(\infty) = \lim_{s \to 0} sF(s)$

9. Initial Value Theorem $\quad f(0+) = \lim_{s \to \infty} sF(s)$