FNU FIJI NATIONAL UNIVERSITY

College of Engineering, Science and Technology (CEST)

EEE743 CONTROL SYSTEMS

Final Examination

Monday 06th June, 2016 0900 - 1210 hours Venue: B314

INSTRUCTIONS TO CANDIDATES

1. Candidates are reminded that they should have no books, notes, paper or other material in their possession unless their use is specifically permitted by "Instructions to Candidates" set out below.
2. Reading time is of 10 minutes duration.
3. Examination time is of 3 hours duration.
4. This paper consists of 8 questions printed on 11 pages.
5. Attempt all 8 questions. Each question may carry a different mark.
6. A Formula Sheet is on page 1.
7. A set of Laplace Transforms Table is attached.
8. The datasheet for the 74LS138 Dual 4-to1 Decoder is also attached.
9. Write your candidate number at the top of each attached sheet.
10. Start each question on a new page.
11. Non-Programmable calculators may be used.
12. Cellphones are not allowed inside the examination venue.

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FORMULA SHEET

1. Integrating Factor $R = e^{\int P(t)dt}$; $Rf(t) = \int RQ(t)dt$
2. $\int u dv = uv - \int v du$
3. $G(s) = \dfrac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$
4. $\%OS = e^{-\left(\frac{\xi\pi}{\sqrt{1-\xi^2}}\right)} \times 100\%$
5. $T_p = \dfrac{\pi}{\omega_n\sqrt{1-\xi^2}}$
6. $T_s = \dfrac{4}{\xi\omega_n}$
7. $s = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
8. $e_n = \sqrt{4kT(BW)R}$
9. $P_n = kT(BW)$
10. $CLTF = \dfrac{G(s)}{1 + G(s)H(s)}$
11. $f(t) = a_0 + \sum_{n=1}^{\infty}\left(a_n \cos\frac{n\pi}{L}t + b_n \sin\frac{n\pi}{L}t\right)$; $a_0 = \frac{1}{T}\int_0^T f(t)dt = \frac{1}{2L}\int_0^{2L} f(t)dt$;

    $a_n = \frac{2}{T}\int_0^T f(t)\cos n\omega t\, dt = \frac{1}{L}\int_0^{2L} f(t)\cos\frac{n\pi}{L}t\, dt$;

    $b_n = \frac{2}{T}\int_0^T f(t)\sin n\omega t\, dt = \frac{1}{L}\int_0^{2L} f(t)\sin\frac{n\pi}{L}t\, dt$
12. $C = 2B\log_2 M$; $C = B\log_2\left(1 + \frac{S}{N}\right)$
13. $q = CV$; $i(t) = \dfrac{v(t)}{R}$; $i(t) = \dfrac{dq(t)}{dt}$; $i(t) = C\dfrac{dv(t)}{dt}$; $i(t) = \dfrac{1}{L}\int v(t)\, dt$; $w_C = \dfrac{1}{2}CV^2$
14. $v(t) = L\dfrac{di(t)}{dt}$; $w_L = \dfrac{1}{2}LI^2$

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QUESTION 1: FIRST ORDER *RC* & *RL* NETWORKS [TOTAL: 14 MARKS]

(a) The switch S1 in the First Order *RL* circuit in Figure 1 is closed at time $t = 0$. It has zero initial conditions.

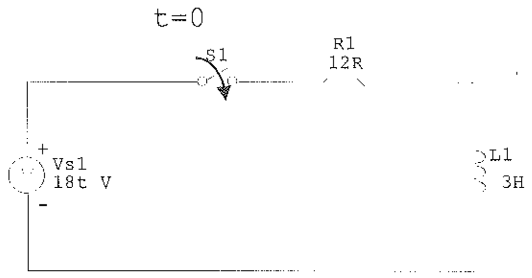

Figure 1: 1^{st} Order *RL* network

(i) Resolve for the step input response solution, $i(t)$, by applying Kirchhoff's Voltage Law (KVL) and using the Integrating Factor method of conventional Calculus. Identify the Transient State. **[5 marks]**

(ii) Resolve for $i(t)$ using Laplace Transforms. **[4 marks]**

(iii) Resolve for the expression for the instantaneous voltage, $v(t)$, across the 3 H inductor. **[2 marks]**

(iv) The step response of a 1^{st} Order System is $C(s) = \frac{5}{s(s+2)}$. Use the method of Poles & Zeros to determine time-domain response. **[3 marks]**

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QUESTION 2: SECOND ORDER SYSTEMS **[TOTAL: 17 MARKS]**

(a) The Transfer Function of an Underdamped system is: $G(s) = \dfrac{25}{s^2 + 6s + 25}$

Find the following:

(i) Peak time (T_p) **[4 marks]**

(ii) Percentage Overshoot **[3 marks]**

(iii) Settling time (T_s) **[2 marks]**

(b) Consider the Series *RLC* circuit zero initial conditions shown in Figure 2. The output is taken across the inductor, *L*. Use Laplace Transforms to derive the solution of q(t). **[6 marks]**

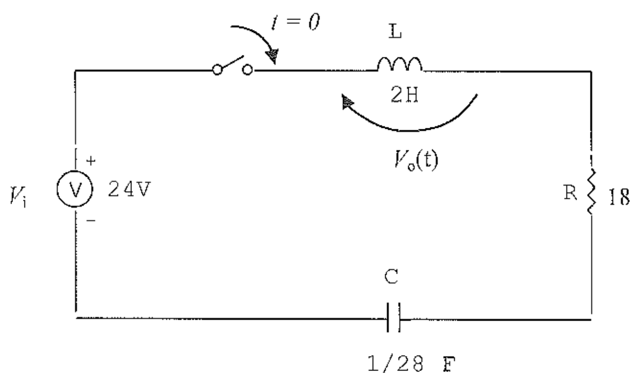

Figure 2: Series *RLC* circuit

(c) Derive the expression for the instantaneous current through the capacitor. **[2 marks]**

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QUESTION 3 : ROOT LOCUS **[TOTAL: 10 MARKS]**

Analyze the system shown in Figure 3.

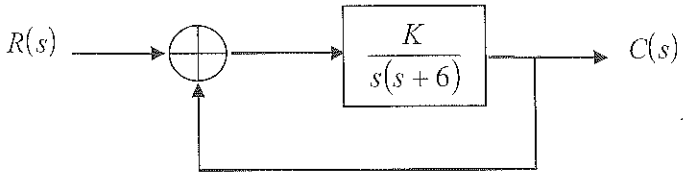


Figure 3: A 2nd Order System

(a) Derive the Characteristic Equation in quadratic form. **[3 marks]**

(b) Sketch the Root Locus. [Hint: Obtain roots for k = 0, 1, 3, 6, 9, 12, 15] **[4 marks]**

(c) From the information provided by the Root Locus, determine the values of k that will cause the system to be Overdamped, Underdamped, and Critically Damped, whilst keeping the system stable. **[3 marks]**

QUESTION 4: LAPLACE TRANSFORMS; UNIT STEP FUNCTION [TOTAL: 12 MARKS]

(a) Solve for the s-domain equivalents of the following functions using the given table of Laplace Transform.

(i) $f(t) = -7\cos 4t + 3e^{2t} - 5t^3$ **[2 marks]**

(ii) $f(t) = 2e^{3t}\sin 5t$ **[2 marks]**

(iii) $f(t) = \begin{cases} (t-4)^3, & t > 4 \\ 0, & t < 4 \end{cases} = u(t-4)(t-4)^3$ **[2 marks]**

(b) Sketch the functions,

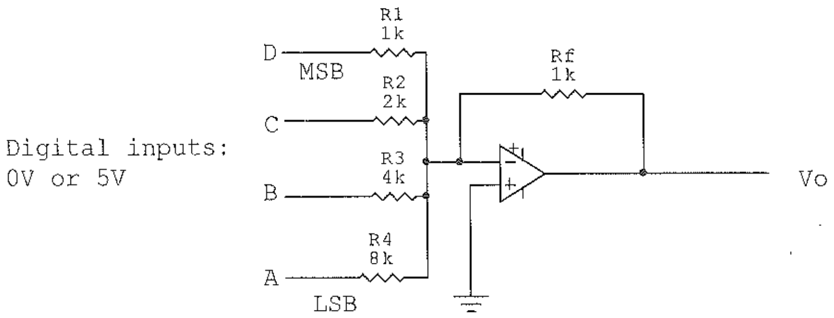
(i) $f(t) = 4u(t) + u(t-7)$ **[2marks]**

(ii) $f(t) = 8u\left(t - \frac{\pi}{2}\right)\sin t, \quad 0 \le t \le 2\pi$ **[2marks]**

(i) $f(t) = u(t).t^3, \quad f(t+2) = f(t); \quad 0 \le t \le 6$ **[2marks]**

QUESTION 5 : SIGNAL CONDITIONING **[TOTAL: 13 MARKS]**

(a) A 4-bit DAC is using an Op-Amp summing amplifier with binary-weighted resistors as shown in **Figure 4**. Assume that **Logic 1 = 12 V** whilst Logic 0 = 0 V.



[5 marks]

Figure 4: 4-bit DAC

Determine the step size (resolution) and find the output voltages for all the combinations of the input code, from DCBA = 0000 to 1111. Tabulate your results as shown in Table 1:

D	C	B	A		V_{out} (v)
0	0	0	0		
0	0	0	1		
0	0	1	0		
0	0	1	1		
0	1	0	0		
0	1	0	1		
0	1	1	0		
0	1	1	1		
1	0	0	0		
1	0	0	1		
1	0	1	0		
1	0	1	1		
1	1	0	0		
1	1	0	1		
1	1	1	0		
1	1	1	1		

Table 1: DAC Table

(b) Describe concisely how the analogue voltage, $V_A = 9$ V, is converted to its digital equivalent by the ADC illustrated in Figure 5. **[4 marks]**

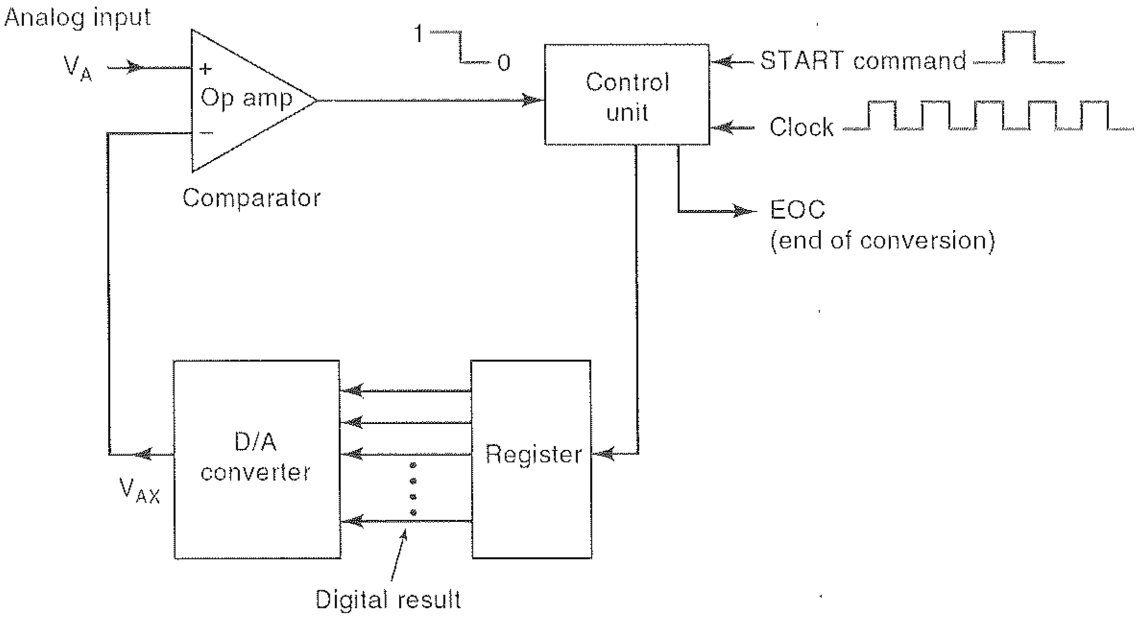


Figure 5: ADC

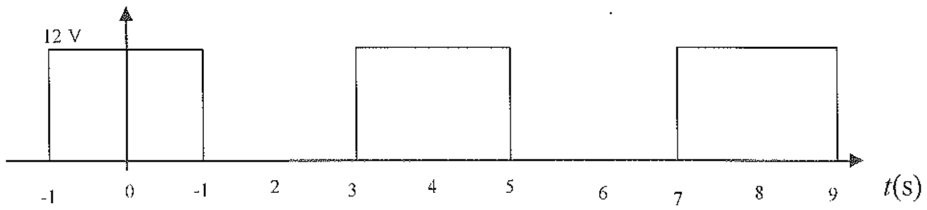
(c) Realize the two mutually exclusive Boolean functions, $f_1(C,B,A) = \sum_m(0,1,6,7)$ and $f_2 = (C,B,A) = \sum_m(1,2,3,5)$ using the **74LS138 1-of-8 Decoder**. Use LEDs as visual indicators for outputs f_1 and f_2. **[10 marks]**

QUESTION 6 [FOURIER SERIES, NOISE, BANDWIDTH] [TOTAL: 22 MARKS]

(a) Refer to the current pulse train shown.

(i) Use the Fourier Series to synthesize the equivalent of the function *i(t)*, shown in **Figure 6**. Determine the first 5 terms. The pulse train represents the function,

$$v(t) = \begin{cases} 12, & -1 \le t < 1 \\ 0, & 1 \le t < 3 \end{cases}; \quad v(t+4) = v(t)$$



[6 marks]

Figure 6: Voltage pulse train

(ii) Determine the amplitude and the frequency for the term when n = 11. **[2 marks]**

(b) A Modem has a bandwidth of 2 MHz, voltage gain of 100 and an input resistance of 10 kΩ. The room temperature is 27 °C. The input audio signal is 12 uV_{rms}. With Boltzmann's constant at 1.38 x 10^{-23} J/K, determine the following

(i) White [Johnson] Noise Power **[3 marks]**

(ii) RMS input noise level **[2 marks]**

(iii) Audio output level **[2 marks]**

(iv) RMS output noise level **[2 marks]**

(b) The bandwidth, *B*, of an Audio Signal is 20 kHz.

(i) Solve for the Maximum Data Transfer Rate (Capacity) using the Nyquist (Hartley) equation when there are 16 signalling levels. Assume a noiseless environment. **[2 marks]**

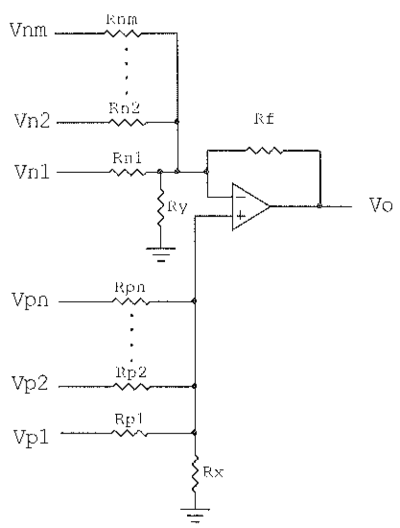
(ii) When the S/N ratio is 64 dB, determine the maximum information rate, *C*, predicted by Shannon's law, $C = B\log_2\left(1+\frac{S}{N}\right)$.

[3 marks]

QUESTION 7 OPERATIONAL AMPLIFIERS – ANALOGUE COMPUTERS [TOTAL: 10 MARKS]

Given the second order differential equation: $v'' = 5v' - v + 4e^{-2t}$. Design an Analogue Computer using Operational amplifiers to solve it. Use integrators whose time constant *RC* = 1. Assume the initial conditions v'(0) = 0 and v(0) = 2 V. Provide a block diagrammatic representation of the circuit first. State any assumptions you make. Do note the General Add-Subtract circuit shown in Figure 7, and the parameters that need to be evaluated.

[10 marks]



$$V_o = \sum_{i=1}^{n} A_i v_{pi} - \sum_{i=1}^{m} B_i v_{ni}$$

where, $A_i = \frac{R_f}{R_{pi}}, B_i = \frac{R_f}{R_{ni}}$

Let $A = \sum A_i, B = \sum B_i$

Let $C = A - B - 1$

If

$$\begin{cases} C \ge 0 & R_x = \infty & R_y = \frac{R_f}{C} \\ C < 0 & R_x = -\frac{R_f}{C} & R_y = \infty \end{cases}$$

Figure 7: General Add-Subtract circuit

QUESTION 8: PARALLEL *RLC* NETWORK [TOTAL: 12 MARKS]

Analyse the Parallel *RLC* network shown in

(a) **Figure 8**.

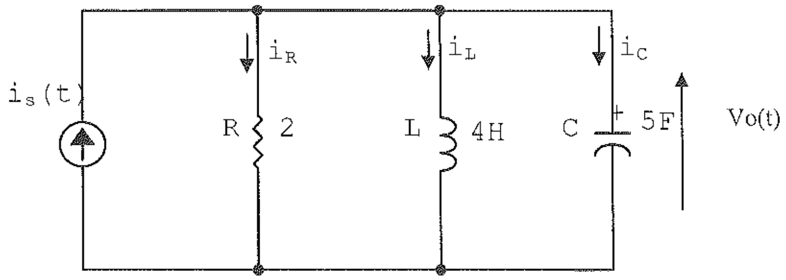


Figure 8: Parallel *RLC* network

(i) Derive the mathematical model in the s-domain. Assume zero initial conditions. **[4 marks]**

(ii) Construct the block diagram of the network, then reduce it to its simplest Closed-loop form using the block reduction technique. **[4 marks]**

(iii) Find the Closed-loop Transfer Function. Represent this via a block diagram. **[4 marks]**

[THE END]

TABLE OF LAPLACE TRANSFORMS

	$F(s) = L[f(t)]$	$f(t),\ t \geq 0$
1.	1	$\delta(t_0)$, Unit Impulse at $t = t_0$
2.	$\frac{1}{s}$	1, Unit Step
3.	$\frac{n!}{s^{n+1}}$	t^n
4.	$\frac{1}{s+a}$	e^{-at}
5.	$\frac{1}{(s+a)^n}$	$\frac{1}{(n-1)!} t^{n-1} e^{-at}$
6.	$\frac{a}{s(s+a)}$	$1 - e^{-at}$
7.	$\frac{1}{(s+a)(s+b)}$	$\frac{1}{(b-a)}\left(e^{-at} - e^{-bt}\right)$
8.	$\frac{s+\alpha}{(s+a)(s+b)}$	$\frac{1}{(b-a)}\left[(\alpha-a)e^{-at} - (\alpha-b)e^{-bt}\right]$
9.	$\frac{ab}{s(s+a)(s+b)}$	$1 - \frac{b}{(b-a)}e^{-at} + \frac{a}{(b-a)}e^{-bt}$
10.	$\frac{1}{(s+a)(s+b)(s+c)}$	$\frac{e^{-at}}{(b-a)(c-a)} + \frac{e^{-bt}}{(c-a)(a-b)} + \frac{e^{-ct}}{(a-c)(b-c)}$
11.	$\frac{s+\alpha}{(s+a)(s+b)(s+c)}$	$\frac{(\alpha-a)e^{-at}}{(b-a)(c-a)} + \frac{(\alpha-b)e^{-bt}}{(c-b)(a-b)} + \frac{(\alpha-c)e^{-ct}}{(a-c)(b-c)}$
12.	$\frac{ab(s+\alpha)}{s(s+a)(s+b)}$	$\alpha - \frac{b(\alpha-a)}{(b-a)}e^{-at} + \frac{a(\alpha-a)}{(b-a)}e^{-bt}$
13.	$\frac{\omega}{s^2+\omega^2}$	$\sin \omega t$
14.	$\frac{s}{s^2+\omega^2}$	$\cos \omega t$
15.	$\frac{s+a}{s^2+\omega^2}$	$\frac{\sqrt{\alpha^2+\omega^2}}{\omega}\sin(\omega t + \phi)$, $\phi = \tan^{-1}\left(\frac{\omega}{\alpha}\right)$
16.	$\frac{\omega}{(s+a)^2+\omega^2}$	$e^{-at} \sin \omega t$

Please turn over...

$F(s) = L[f(t)]$	$f(t), \quad t \geq 0$
17. $\dfrac{(s+a)}{(s+a)^2+\omega^2}$	$e^{-at}\cos\omega t$
18. $\dfrac{s+\alpha}{(s+a)^2+\omega^2}$	$\dfrac{1}{\omega}\left[(\alpha-a)^2+\omega^2\right]^{\frac{1}{2}} e^{-at}\sin(\omega t+\phi)$, $\phi = \tan^{-1}\left(\dfrac{\omega}{\alpha-a}\right)$
19. $\dfrac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$	$\dfrac{\omega_n}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t}\sin\omega_n\sqrt{1-\zeta^2}\,t,\ \zeta<1$
20. $\dfrac{1}{s\left[(s+a)^2+\omega^2\right]}$	$\dfrac{1}{a^2+\omega^2}+\dfrac{1}{\omega\sqrt{a^2+\omega^2}} e^{-at}\sin(\omega t-\phi)$, $\phi = \tan^{-1}\left(\dfrac{\omega}{-a}\right)$
21. $\dfrac{\omega_n^2}{s\left(s^2+2\zeta\omega_n s+\omega_n^2\right)}$	$1-\dfrac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t}\sin\left(\omega_n\sqrt{1-\zeta^2}\,t+\phi\right)$, $\phi=\cos^{-1}\zeta, \zeta<1$
22. $\dfrac{(s+\alpha)}{s\left[(s+a)^2+\omega^2\right]}$	$\dfrac{\alpha}{a^2+\omega^2}+\dfrac{1}{\omega}\left[\dfrac{(\alpha-a)^2+\omega^2}{a^2+\omega^2}\right]^{\frac{1}{2}} e^{-at}\sin(\omega t+\phi)$, $\phi=\tan^{-1}\left(\dfrac{\omega}{\alpha-a}\right)-\tan^{-1}\left(\dfrac{\omega}{-a}\right)$
23. $\dfrac{1}{(s+c)\left[(s+a)^2+\omega^2\right]}$	$\dfrac{e^{-ct}}{(c-a)^2+\omega^2}+\dfrac{e^{-at}\sin(\omega t+\phi)}{\omega\left[(c-a)^2+\omega^2\right]^{\frac{1}{2}}}$, $\phi=\tan^{-1}\left(\dfrac{\omega}{c-a}\right)$

24. $\dfrac{a}{s^2 - a^2}$	$\sinh at$
25. $\dfrac{s}{s^2 - a^2}$	$\cosh at$
26. $\dfrac{\omega^2}{s(s^2 + \omega^2)}$	$1 - \cos \omega t$
27. $\dfrac{\omega^3}{s^2(s^2 + \omega^2)}$	$\omega t - \sin \omega t$
28. $\dfrac{2\omega^3}{(s^2 + \omega^2)^2}$	$\sin \omega t - \omega t \cos \omega t$

	$F(s) = L[f(t)]$	$f(t),\ t \geq 0$
29.	$\dfrac{s}{(s^2+\omega^2)^2}$	$\dfrac{t}{2\omega}\sin\omega t$
30.	$\dfrac{s^2}{(s^2+\omega^2)^2}$	$\dfrac{1}{2\omega}(\sin\omega t + \omega t\cos\omega t)$
31.	$\dfrac{s}{(s^2+a^2)(s^2+b^2)}\quad (a^2 \neq b^2)$	$\dfrac{1}{b^2-a^2}(\cos at - \cos bt)$

LAPLACE TRANSFORM OF DERIVATIVES

32.	$sF(s) - f(0)$	$f'(t)$, First Derivative
33.	$s^2F(s) - sf(0) - f'(0)$	$f''(t)$, Second Derivative

GENERAL PROPERTIES OF LAPLACE TRANSFORMS

1. Linearity $\quad L[af_1(t)+bf_2(t)] = aF_1(s)+bF_2(s)$

2. Scaling $\quad L[f(at)] = \dfrac{1}{a}F\left(\dfrac{s}{a}\right)$

3. Transform of a Derivative

$$L\left[f^{(n)}(t)\right] = s^n F(s) - s^{n-1}f(0) - s^{n-2}f'(0) - \ldots - sf^{(n-2)}(0) - f^{(n-1)}(0)$$

4. First Shifting Theorem $\quad L\left[e^{at}f(t)\right] = F(s-a)$
   (Complex Translation; s-shifting)

5. Second Shifting Theorem $\quad L[u(t-a)f(t-a)] = e^{-as}F(s)$
   (Real Translation; t—shifting)

6. Integration $\quad L\left[\int_0^t f(\tau)d\tau\right] = \dfrac{1}{s}F(s)$

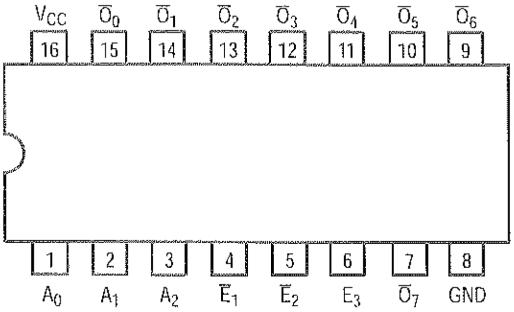
7. Complex Differentiation $\quad L[tf(t)] = -\dfrac{dF(s)}{ds}$

8. Final Value Theorem $\quad f(\infty) = \lim_{s\to 0} sF(s)$

9. Initial Value Theorem $\quad f(0+) = \lim_{s\to\infty} sF(s)$

SN74LS138

CONNECTION DIAGRAM DIP (TOP VIEW)

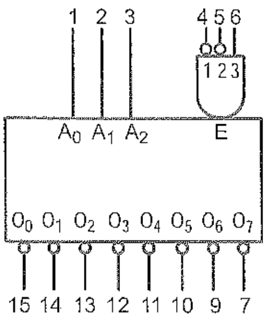


NOTE:
The Flatpak version has the same pinouts (Connection Diagram) as the Dual In-Line Package.

PIN NAMES		**LOADING** (Note a) HIGH	LOW
$A_0 - A_2$	Address Inputs	0.5 U.L.	0.25 U.L.
$\overline{E}_1$, $\overline{E}_2$	Enable (Active LOW) Inputs	0.5 U.L.	0.25 U.L.
E_3	Enable (Active HIGH) Input	0.5 U.L.	0.25 U.L.
$\overline{O}_0 - \overline{O}_7$	Active LOW Outputs	10 U.L.	5 U.L.

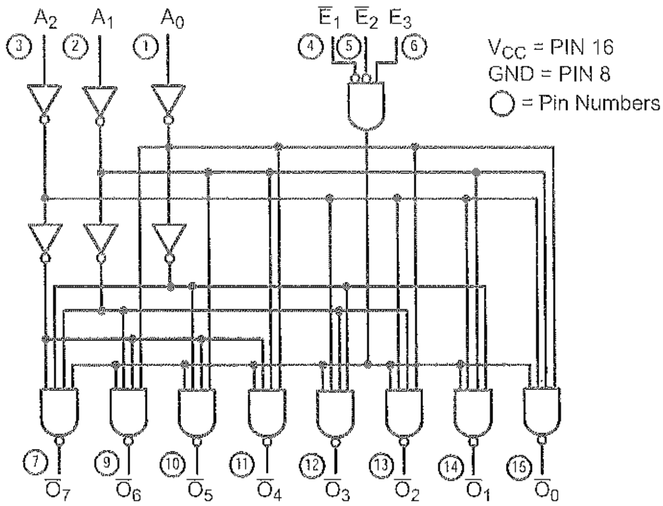
NOTES:
a) 1 TTL Unit Load (U.L.) = 40 µA HIGH/1.6 mA LOW.

LOGIC SYMBOL



V_{CC} = PIN 16
GND = PIN 8

LOGIC DIAGRAM



FUNCTIONAL DESCRIPTION

The LS138 is a high speed 1-of-8 Decoder/Demultiplexer fabricated with the low power Schottky barrier diode process. The decoder accepts three binary weighted inputs (A_0, A_1, A_2) and when enabled provides eight mutually exclusive active LOW Outputs ($\overline{O}_0$–$\overline{O}_7$). The LS138 features three Enable inputs, two active LOW ($\overline{E}_1$, $\overline{E}_2$) and one active HIGH (E_3). All outputs will be HIGH unless $\overline{E}_1$ and $\overline{E}_2$ are LOW and E_3 is HIGH. This multiple enable function allows easy parallel expansion of the device to a 1-of-32 (5 lines to 32 lines) decoder with just four LS138s and one inverter. (See Figure a.)

The LS138 can be used as an 8-output demultiplexer by using one of the active LOW Enable inputs as the data input and the other Enable inputs as strobes. The Enable inputs which are not used must be permanently tied to their appropriate active HIGH or active LOW state.

TRUTH TABLE

INPUTS						OUTPUTS							
$\overline{E}_1$	$\overline{E}_2$	E_3	A_0	A_1	A_2	$\overline{O}_0$	$\overline{O}_1$	$\overline{O}_2$	$\overline{O}_3$	$\overline{O}_4$	$\overline{O}_5$	$\overline{O}_6$	$\overline{O}_7$
H	X	X	X	X	X	H	H	H	H	H	H	H	H
X	H	X	X	X	X	H	H	H	H	H	H	H	H
X	X	L	X	X	X	H	H	H	H	H	H	H	H
L	L	H	L	L	L	L	H	H	H	H	H	H	H
L	L	H	H	L	L	H	L	H	H	H	H	H	H
L	L	H	L	H	L	H	H	L	H	H	H	H	H
L	L	H	H	H	L	H	H	H	L	H	H	H	H
L	L	H	L	L	H	H	H	H	H	L	H	H	H
L	L	H	H	L	H	H	H	H	H	H	L	H	H
L	L	H	L	H	H	H	H	H	H	H	H	L	H
L	L	H	H	H	H	H	H	H	H	H	H	H	L

H = HIGH Voltage Level
L = LOW Voltage Level
X = Don't Care

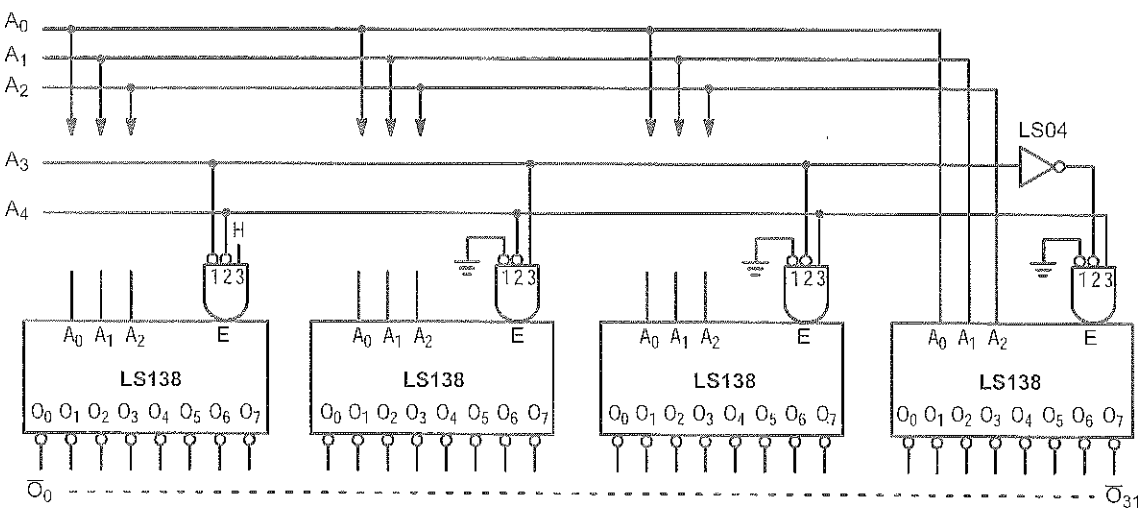


Figure a