FIJI NATIONAL UNIVERSITY

College of Engineering, Science and Technology

School of Electrical &Electronics Engineering

EEE 694 – Engineering Mathematics III

Semester I, 2016

FINAL EXAMINATION

Programme: Bachelor of Engineering, 2nd year

Time Allowed: 3 HOURS 10 MINS

Time: Date:

Instructions to Students:

1. There are thirteen (13) questions in the paper. Answer any ten (10) questions.
2. You are allowed 10 minutes extra reading time during which you are NOT allowed to write.
3. This exam is worth 50% of your overall marks.
4. Answer questions neatly on a new page and clearly number the question attempted.
5. Students may use a calculator, provided it is silent and non-programmable.
6. If you use extra sheets of paper, attach it securely to the answer booklet.

Question 1

(a) Verify by substitution that the functions $y = x^2$ and $y = 1$ are solutions of the nonhomogeneous linear ordinary differential equation $y''y - xy' = 0$, but their sum is a not a solution.

(b) Solve the ordinary differential equation $2xyy' = y^2 - x^2$

[6+4]

Question 2

(a) Verify by substitution that the functions $y_1 = e^x$ and $y_2 = e^{-x}$ are solutions of the ordinary differential equation $y'' - y = 0$. Then solve the initial value problem $y'' - y = 0, y(0) = 6, y'(0) = -2$

(b) Solve the initial value problem $y'' + y' - 6y = 0, y(0) = 10, y'(0) = 0$

[5+5]

Question 3

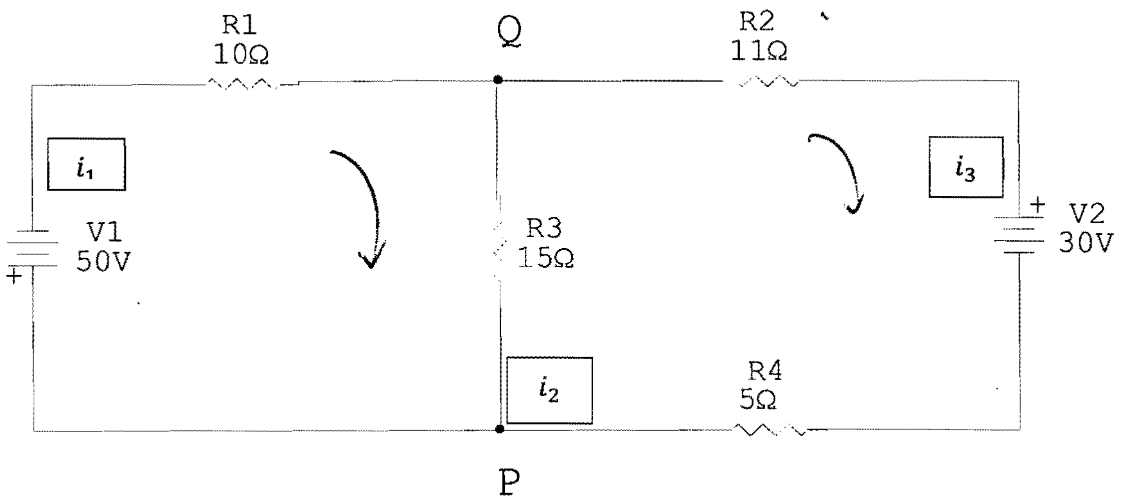
(a) Solve $y'' + 5y' + 4y = 10e^{-3x}$

(b) Solve the initial value problem $y'' + 4y = -12sin2x, y(0) = 1.8, y'(0) = 5.0$

[6+4]

Question 4

Find the currents $i_1, i_2, and\ i_3$ in the following electrical circuit using Gauss elimination method. Use Kirchhoff's Voltage and Current Laws for modeling.



[10]

Question 5

(a) Solve the ordinary differential equation $3x(xy - 2)dx + (x^3 + 2y)dy = 0$

(b) Solve the initial value problem $y' + xy = xy^{-1}, y(0) = 3$

[4+6]

Question 6

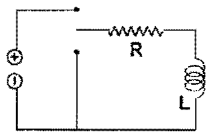
(a) Let $A = \begin{pmatrix} -1 & 2 & 1 & 3 & 4 \\ 0 & 0 & 2 & -1 & 0 \end{pmatrix}$. Let $T: R^5 \to R^2$ be the linear transformation $T(x) = Ax$. Then compute $T(1,0,-1,3,0)$.

(b) Find the standard matrix for the linear transformation $T: R^3 \to R^3$ defined by $T(x, y, z) = (2x - 5y + 3z, 2x + 7y + 4z, -5x + 2y + 7z)$.

[5+5]

Question 7

(a) Suppose that in the simple circuit as shown in the figure below the resistance is 10 Ω and the inductance is 5H. If a battery gives a constant voltage of 75V and the switch is closed when t = 0 so the current starts with I(0) = 0, find:



(i) current I(t),
(ii) the current after 5 sec, and
(iii) the limiting value of the current. [5+2+3]

Question 8

Verify this for A and $P^{-1}AP$. Find eigen vectors y of P. Show that x= Py are eigen vectors of A,

where $A = \begin{pmatrix} 0 & 0 & 2 \\ 0 & 3 & 2 \\ 1 & 0 & 1 \end{pmatrix}$ and $P = \begin{pmatrix} 2 & 0 & 3 \\ 0 & 1 & 0 \\ 3 & 0 & 5 \end{pmatrix}$ [10]

Question 9

Find the eigenvalues and corresponding eigen vectors of $A = \begin{pmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{pmatrix}$

[10]

Question 10

(a) Find a real general solution of the Euler-Cauchy equation $x^2y'' - 5xy' + 9y = 0$.

(b) Solve the initial value problem $y'' + 3y = 18x^2, y(0) = -3, y'(0) = 0$

[5+5]

Question 11

The linear transformation $T: R^3 \rightarrow R^3$ is defined by

$T(x, y, z) = (-x + y + 2z, 3x - y + z, -x + 3y + 4z)$. Show that T is invertible, and find its inverse.

[10]

Question 12

Find the steady – state current I(t) in an RLC-circuit with R = 50Ω, L = 30H, C = 0.025F, which is connected to a source of EMF E(t) = 200 sin4t Volts . Assuming zero initial current and charge.

[10]

Question 13

The linear transformation T is given by T(x) = Ax, where $A = \begin{pmatrix} 0 & 4 & -1 & 5 \\ -4 & 0 & 3 & -2 \\ 1 & -3 & 0 & 1 \\ -5 & 2 & -1 & 0 \end{pmatrix}$.

Find:
(i) Rank (T)
(ii) Nullity (T)

[10]

-THE END-