FIJI NATIONAL UNIVERSITY

College of Engineering, Science and Technology (CEST)

EEE743 CONTROL SYSTEMS

Bachelor of Engineering (Electrical & Electronics)
Supplementary Examination

Friday 05th August, 2016   0900 - 1210 hours   Venue: C106

INSTRUCTIONS TO CANDIDATES

1. Candidates are reminded that they should have no books, notes, paper or other material in their possession unless their use is specifically permitted by "Instructions to Candidates" set out below.
2. Reading time is of 10 minutes duration.
3. Examination time is of 3 hours duration.
4. This paper consists of 8 questions printed on 11 pages.
5. Attempt all 8 questions. Each question may carry a different mark.
6. A Formula Sheet is on page 1.
7. A set of Laplace Transforms Table is attached.
8. The datasheet for the 74LS153 Multiplexer is on page 9.
9. Write your candidate number at the top of each attached sheet.
10. Start each question on a new page.
11. Non-Programmable Calculators may be used.
12. Mobile phones are not allowed inside the examination venue.

EEE743 SUPPLEMENTARY EXAMINATION 2016

FORMULA SHEET

1. Integrating Factor $R = e^{\int P(t)dt}$; $Rf(t) = \int RQ(t)dt$
2. $\int u dv = uv - \int v du$
3. $G(s) = \dfrac{\omega_n^{\ 2}}{s^2 + 2\xi\omega_n s + \omega_n^{\ 2}}$
4. $\%OS = e^{-\left(\frac{\xi\pi}{\sqrt{1-\xi^2}}\right)} \times 100\%$
5. $T_p = \dfrac{\pi}{\omega_n\sqrt{1-\xi^2}}$
6. $T_s = \dfrac{4}{\xi\omega_n}$
7. $s = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
8. $e_n = \sqrt{4kT(BW)R}$
9. $P_n = kT(BW)$
10. $CLTF = \dfrac{G(s)}{1+G(s)H(s)}$
11. $f(t) = a_0 + \sum_{n=1}^{\infty}\left(a_n \cos\frac{n\pi}{L}t + b_n \sin\frac{n\pi}{L}t\right)$; $a_0 = \frac{1}{T}\int_0^T f(t)dt = \frac{1}{2L}\int_0^{2L} f(t)dt$;

    $a_n = \frac{2}{T}\int_0^T f(t)\cos n\omega t\, dt = \frac{1}{L}\int_0^{2L} f(t)\cos\frac{n\pi}{L}t\, dt$;

    $b_n = \frac{2}{T}\int_0^T f(t)\sin n\omega t\, dt = \frac{1}{L}\int_0^{2L} f(t)\sin\frac{n\pi}{L}t\, dt$
12. $C = 2B\log_2 M$; $C = B\log_2\left(1+\frac{S}{N}\right)$
13. $q = CV$; $i(t) = \frac{dq(t)}{dt}$; $i(t) = C\frac{dv(t)}{dt}$ $w_C = \frac{1}{2}CV^2$
14. $v(t) = L\frac{di(t)}{dt}$; $w_L = \frac{1}{2}LI^2$

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QUESTION 1: FIRST ORDER *RC* & *RL* NETWORKS [TOTAL: 14 MARKS]

(a) The 1st Order *RC* circuit in Figure 1 has zero initial conditions. The switch S1 is closed at time $t = 0$.

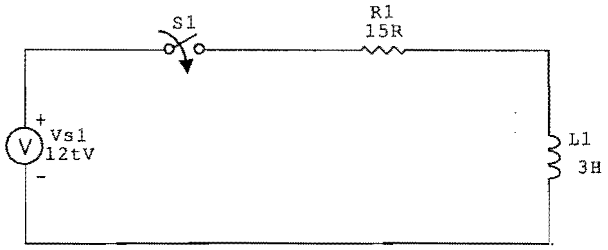


Figure 1: 1st Order *RC* network

(i) Determine the step input response solution, $i(t)$, by applying Kirchhoff's Voltage Law (KVL) and using the Integrating Factor method of conventional Calculus. Identify the Transient State. **[5 marks]**

(ii) Resolve for $i(t)$ using Laplace Transforms. **[4 marks]**

(iii) Express the instantaneous voltage, $v(t)$, across the 15Ω resistor. **[2 marks]**

(iv) The step response of a First Order System is $C(s) = \frac{4}{s(s+3)}$. Use the method of Poles & Zeros to determine time-domain response.

[3 marks]

QUESTION 2: SECOND ORDER SYSTEMS **[TOTAL: 17 MARKS]**

(a) Given an Underdamped system, find the following if the Transfer Function is

$$G(s) = \frac{81}{s^2 + 12s + 81}$$

(i) Peak time (T_p) **[4 marks]**

(ii) Percentage Overshoot **[3 marks]**

(iii) Settling time (T_s) **[2 marks]**

(b) Refer to the Series *RLC* circuit shown in **Figure 2**. The output is taken across the inductor, *L*. Use Laplace Transforms to derive the solution of *q(t)*. Assume zero initial conditions.

[6 marks]

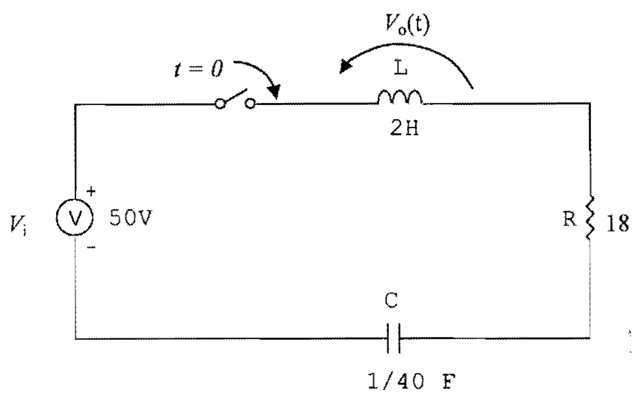


Figure 2: Series *RLC* circuit

(c) Derive the expression for the instantaneous current through the capacitor. **[2 marks]**

QUESTION 3 : ROOT LOCUS [TOTAL: 10 MARKS]

Consider the system shown in Figure 3.

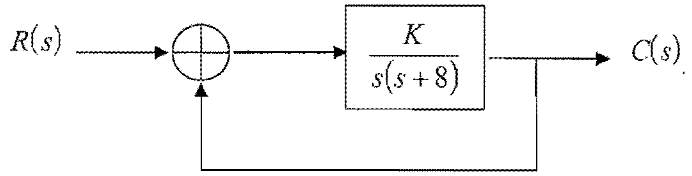


Figure 3: A Second Order System

(a) Derive the Characteristic Equation in quadratic form. **[3 marks]**

(b) Sketch the Root Locus. [Hint: Obtain roots for k = 0, 4, 8, 12, 16, 20, 24] **[4 marks]**

(c) From the information provided by the Root Locus, determine the values of k that will cause the system to be Overdamped, Underdamped, and Critically Damped, whilst keeping the system stable. **[3 marks]**

QUESTION 4: LAPLACE TRANSFORMS; UNIT STEP FUNCTION [TOTAL: 12 MARKS]

(a) Determine the s-domain equivalents of the following functions using the given table of Laplace Transform.

(i) $f(t) = 5e^{-3t} + 3t^5 - 2\cosh 3t$ **[2 marks]**

(ii) $f(t) = 7e^{3t}\cos 9t$ **[2 marks]**

(iii) $f(t) = \begin{Bmatrix} 0, & t < 8 \\ (t-8)^4 & t > 8 \end{Bmatrix} = u(t-8)(t-8)^4$ **[2 marks]**

(b) Sketch the functions,

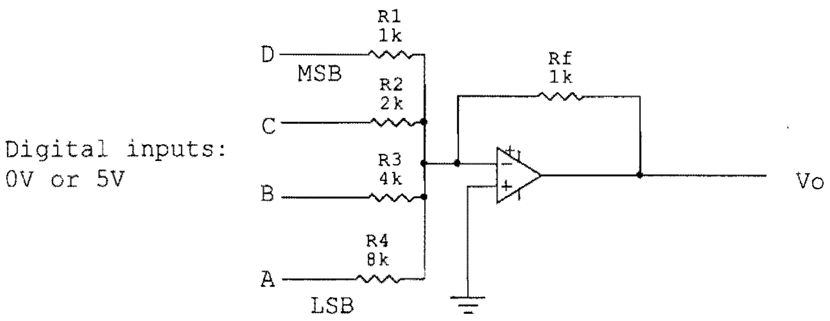
(i) $f(t) = 7u(t) + u(t-10)$ **[2marks]**

(ii) $f(t) = 12u\left(t - \frac{\pi}{2}\right)\sin t, \quad 0 \le t \le 2\pi$ **[2marks]**

(iii) $f(t) = u(t).t^2, \quad f(t+2) = f(t); \quad 0 \le t \le 6$ **[2marks]**

QUESTION 5 : SIGNAL CONDITIONING **[TOTAL: 13 MARKS]**

(a) Consider a 4-bit DAC using an op-amp summing amplifier with binary-weighted resistors as shown in Figure 1. Assume that **Logic 1 = 10V** whilst Logic 0 = 0V.



[5 marks]

Figure 4: 4-bit DAC

Determine the step size (resolution) and find the output voltages for all the combinations of the input code, from DCBA = 0000 to 1111. Tabulate your results as shown in Table 1:

D	C	B	A		V_{out} (v)
0	0	0	0		
0	0	0	1		
0	0	1	0		
0	0	1	1		
0	1	0	0		
0	1	0	1		
0	1	1	0		
0	1	1	1		
1	0	0	0		
1	0	0	1		
1	0	1	0		
1	0	1	1		
1	1	0	0		
1	1	0	1		
1	1	1	0		
1	1	1	1		

Table 1: DAC Table

(b) Describe concisely how the analogue voltage, $V_A = 7V$, is converted to its digital equivalent by the ADC illustrated in

(c) Figure 5. **[4 marks]**

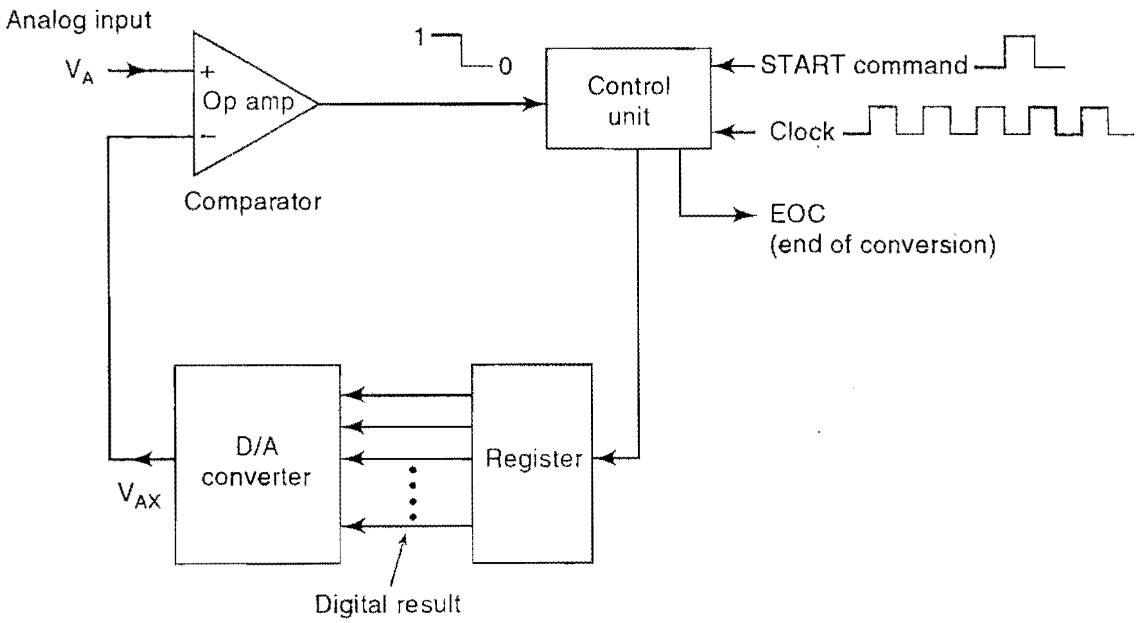
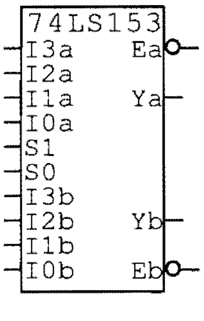


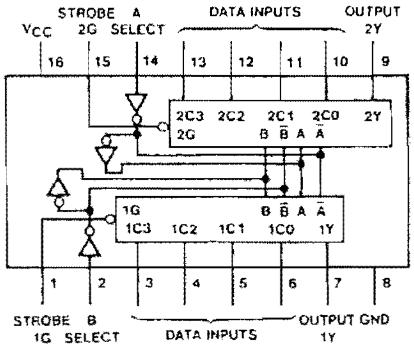
Figure 5: ADC

(d) Two mutually exclusive control circuits are described by the Boolean functions, $f_1(C,B,A) = \sum_m(1, 2, 5, 6, 7)$ and $f_2 = (C,B,A) = \sum_m(0, 1, 3, 4)$. Realize the Boolean functions using the **74LS153** Dual 4-to-1 **Multiplexer,** where only one function is active at a time, i.e. f_1 and f_2 are mutually exclusive. Give explanations where appropriate. Data sheet for the 74LS153 is given below. **[4 marks]**

Figure 6: 74LS153 Datasheet



Connection Diagram



Function Table

Select Inputs		Data Inputs				Strobe	Output
B	A	C0	C1	C2	C3	G	Y
X	X	X	X	X	X	H	L
L	L	L	X	X	X	L	L
L	L	H	X	X	X	L	H
L	H	X	L	X	X	L	L
L	H	X	H	X	X	L	H
H	L	X	X	L	X	L	L
H	L	X	X	H	X	L	H
H	H	X	X	X	L	L	L
H	H	X	X	X	H	L	H

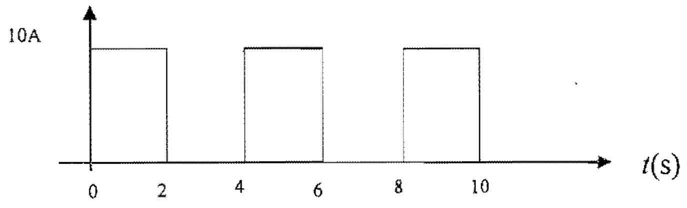
Select inputs A and B are common to both sections
H = HIGH Level
L = LOW Level
X = Don't Care

QUESTION 6 [FOURIER SERIES, NOISE, BANDWIDTH] [TOTAL: 22 MARKS]

(a) Refer to the current pulse train shown.

(i) Use the Fourier Series to synthesize the equivalent of the function *i(t)*, shown in Figure 7. Determine the first 5 terms. The pulse train represents the function,

$$i(t) = \begin{cases} 10, & 0 \le t < 2 \\ 0, & 2 \le t < 4 \end{cases}; \quad i(t+4) = i(t)$$



[6 marks]

Figure 7: Current pulse train

(ii) Predict the amplitude and the frequency for the term when n = 11. **[2 marks]**

(b) A Karaoke system has a bandwidth of 1 MHz, voltage gain of 150 and an input resistance of 5 kΩ. The room temperature is 27 °C. The singers produce an input audio signal of 4 uV_{rms}. With Boltzmann's constant at 1.38×10^{-23} J/K, determine the following

(i) White [Johnson] Noise Power **[3 marks]**

(ii) RMS input noise level **[2 marks]**

(iii) Audio output level **[2 marks]**

(iv) RMS output noise level **[2 marks]**

(b) The bandwidth, *B*, of an Audio Recording studio is 20 kHz.

(i) Calculate the Maximum Data Transfer Rate (Capacity) using the Nyquist (Hartley) equation. There are 8 signalling levels and assume a noiseless environment. **[2 marks]**

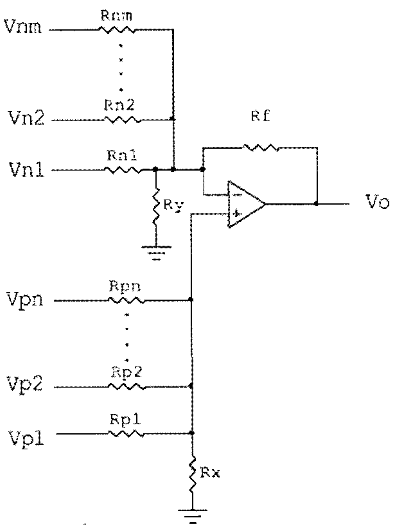
(ii) If the S/N ratio is 50 dB, determine the maximum information rate, *C*, predicted by Shannon's law, $C = B\log_2\left(1+\frac{S}{N}\right)$.

[3 marks]

QUESTION 7 OPERATIONAL AMPLIFIERS – ANALOGUE COMPUTERS [TOTAL: 10 MARKS]

Design an Analogue Computer using Operational amplifiers to solve the second order differential equation, $m'' = -6m' + 3m - 4\sin 2t$. Use integrators whose time constant $RC = 1$. Assume the initial conditions m'(0) = 1 and m(0) = 0.5 V. Provide a block diagrammatic representation of the circuit first. State any assumptions you make. Do note the General Add-Subtract circuit shown in Figure 8, and the parameters that need to be evaluated.

[10 marks]



$$V_o = \sum_{i=1}^{n} A_i v_{pi} - \sum_{i=1}^{m} B_i v_{ni}$$

where. $A_i = \frac{R_f}{R_{pi}}, B_i = \frac{R_f}{R_{ni}}$

Let $A = \sum A_i, B = \sum B_i$

Let $C = A - B - 1$

If

$C > 0$ $R_x = \infty$ $R_y = \frac{R_f}{C}$

$C < 0$ $R_x = -\frac{R_f}{C}$ $R_y = \infty$

Figure 8: General Add-Subtract circuit

QUESTION 8: PARALLEL *RLC* NETWORK **[TOTAL: 12 MARKS]**

(a) Analyse the Parallel *RLC* network shown.

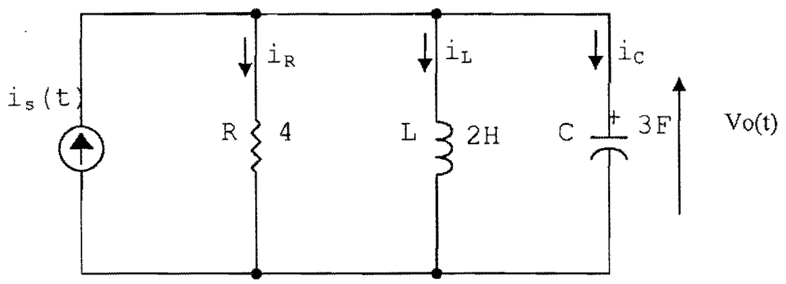


Figure 9: Parallel *RLC* network

(i) Derive the mathematical model in the s-domain. Assume zero initial conditions. **[4 marks]**

(ii) Construct the block diagram of the network, then reduce it to its simplest Closed-loop form using the block reduction technique. **[4 marks]**

(iii) Find the Closed-loop Transfer Function. Represent this via a block diagram. **[4 marks]**

[THE END]

TABLE OF LAPLACE TRANSFORMS

	$F(s)=L[f(t)]$	$f(t),\ t \geq 0$
1.	1	$\delta(t_0)$, Unit Impulse at $t = t_0$
2.	$\frac{1}{s}$	1, Unit Step
3.	$\frac{n!}{s^{n+1}}$	t^n
4.	$\frac{1}{s+a}$	e^{-at}
5.	$\frac{1}{(s+a)^n}$	$\frac{1}{(n-1)!}t^{n-1}e^{-at}$
6.	$\frac{a}{s(s+a)}$	$1-e^{-at}$
7.	$\frac{1}{(s+a)(s+b)}$	$\frac{1}{(b-a)}\left(e^{-at}-e^{-bt}\right)$
8.	$\frac{s+\alpha}{(s+a)(s+b)}$	$\frac{1}{(b-a)}\left[(\alpha-a)e^{-at}-(\alpha-b)e^{-bt}\right]$
9.	$\frac{ab}{s(s+a)(s+b)}$	$1-\frac{b}{(b-a)}e^{-at}+\frac{a}{(b-a)}e^{-bt}$
10.	$\frac{1}{(s+a)(s+b)(s+c)}$	$\frac{e^{-at}}{(b-a)(c-a)}+\frac{e^{-bt}}{(c-a)(a-b)}+\frac{e^{-ct}}{(a-c)(b-c)}$
11.	$\frac{s+\alpha}{(s+a)(s+b)(s+c)}$	$\frac{(\alpha-a)e^{-at}}{(b-a)(c-a)}+\frac{(\alpha-b)e^{-bt}}{(c-b)(a-b)}+\frac{(\alpha-c)e^{-ct}}{(a-c)(b-c)}$
12.	$\frac{ab(s+\alpha)}{s(s+a)(s+b)}$	$\alpha-\frac{b(\alpha-a)}{(b-a)}e^{-at}+\frac{a(\alpha-a)}{(b-a)}e^{-bt}$
13.	$\frac{\omega}{s^2+\omega^2}$	$\sin \omega t$
14.	$\frac{s}{s^2+\omega^2}$	$\cos \omega t$
15.	$\frac{s+a}{s^2+\omega^2}$	$\frac{\sqrt{\alpha^2+\omega^2}}{\omega}\sin(\omega t+\phi)$, $\phi=\tan^{-1}\left(\frac{\omega}{\alpha}\right)$
16.	$\frac{\omega}{(s+a)^2+\omega^2}$	$e^{-at}\sin \omega t$

Please turn over...

$F(s) = \mathcal{L}[f(t)]$	$f(t), \quad t \geq 0$
17. $\dfrac{(s+a)}{(s+a)^2+\omega^2}$	$e^{-at}\cos\omega t$
18. $\dfrac{s+\alpha}{(s+a)^2+\omega^2}$	$\dfrac{1}{\omega}\left[(\alpha-a)^2+\omega^2\right]^{\frac{1}{2}} e^{-at}\sin(\omega t+\phi)$, $\phi = \tan^{-1}\left(\dfrac{\omega}{\alpha-a}\right)$
19. $\dfrac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$	$\dfrac{\omega_n}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t}\sin\omega_n\sqrt{1-\zeta^2}\,t, \ \zeta<1$
20. $\dfrac{1}{s\left[(s+a)^2+\omega^2\right]}$	$\dfrac{1}{a^2+\omega^2}+\dfrac{1}{\omega\sqrt{a^2+\omega^2}} e^{-at}\sin(\omega t-\phi)$, $\phi = \tan^{-1}\left(\dfrac{\omega}{-a}\right)$
21. $\dfrac{\omega_n^2}{s\left(s^2+2\zeta\omega_n s+\omega_n^2\right)}$	$1-\dfrac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t}\sin\left(\omega_n\sqrt{1-\zeta^2}\,t+\phi\right)$, $\phi = \cos^{-1}\zeta, \zeta<1$
22. $\dfrac{(s+\alpha)}{s\left[(s+a)^2+\omega^2\right]}$	$\dfrac{\alpha}{a^2+\omega^2}+\dfrac{1}{\omega}\left[\dfrac{(\alpha-a)^2+\omega^2}{a^2+\omega^2}\right]^{\frac{1}{2}} e^{-at}\sin(\omega t+\phi)$, $\phi = \tan^{-1}\left(\dfrac{\omega}{\alpha-a}\right)-\tan^{-1}\left(\dfrac{\omega}{-a}\right)$
23. $\dfrac{1}{(s+c)\left[(s+a)^2+\omega^2\right]}$	$\dfrac{e^{-ct}}{(c-a)^2+\omega^2}+\dfrac{e^{-at}\sin(\omega t+\phi)}{\omega\left[(c-a)^2+\omega^2\right]^{\frac{1}{2}}}$, $\phi = \tan^{-1}\left(\dfrac{\omega}{c-a}\right)$

24. $\frac{a}{s^2-a^2}$	$\sinh at$
25. $\frac{s}{s^2-a^2}$	$\cosh at$
26. $\frac{\omega^2}{s(s^2+\omega^2)}$	$1-\cos \omega t$
27. $\frac{\omega^3}{s^2(s^2+\omega^2)}$	$\omega t - \sin \omega t$
28. $\frac{2\omega^3}{(s^2+\omega^2)^2}$	$\sin \omega t - \omega t \cos \omega t$

	$F(s) = L[f(t)]$		$f(t), \ t \geq 0$
29.	$\dfrac{s}{\left(s^2+\omega^2\right)^2}$		$\dfrac{t}{2\omega}\sin \omega t$
30.	$\dfrac{s^2}{\left(s^2+\omega^2\right)^2}$		$\dfrac{1}{2\omega}(\sin \omega t + \omega t \cos \omega t)$
31.	$\dfrac{s}{\left(s^2+a^2\right)\left(s^2+b^2\right)}$	$\left(a^2 \neq b^2\right)$	$\dfrac{1}{b^2-a^2}(\cos at - \cos bt)$

LAPLACE TRANSFORM OF DERIVATIVES

32.	$sF(s) - f(0)$	$f'(t)$, First Derivative
33.	$s^2F(s) - sf(0) - f'(0)$	$f''(t)$, Second Derivative

GENERAL PROPERTIES OF LAPLACE TRANSFORMS

1. Linearity $\quad L[af_1(t) + bf_2(t)] = aF_1(s) + bF_2(s)$

2. Scaling $\quad L[f(at)] = \dfrac{1}{a}F\left(\dfrac{s}{a}\right)$

3. Transform of a Derivative

$$L\left[f^{(n)}(t)\right] = s^nF(s) - s^{n-1}f(0) - s^{n-2}f'(0) - \ldots - sf^{(n-2)}(0) - f^{(n-1)}(0)$$

4. First Shifting Theorem $\quad L\left[e^{at}f(t)\right] = F(s-a)$
   (Complex Translation; s-shifting)

5. Second Shifting Theorem $L[u(t-a)f(t-a)] = e^{-as}F(s)$
   (Real Translation; t—shifting)

6. Integration $\quad L\left[\int_0^t f(\tau)d\tau\right] = \dfrac{1}{s}F(s)$

7. Complex Differentiation $\quad L[tf(t)] = -\dfrac{dF(s)}{ds}$

8. Final Value Theorem $\quad f(\infty) = \lim\limits_{s \to 0} sF(s)$

9. Initial Value Theorem $\quad f(0+) = \lim\limits_{s \to \infty} sF(s)$