FIJI NATIONAL UNIVERSITY

College of Engineering, Science and Technology

School of Mathematical & Computing Sciences

EEE 605- Mathematics for Engineers

Semester I

FINAL EXAMINATION

2015

Programme: Advanced Diploma in Electrical & Electronics Engineering

Time Allowed: 3 Hours 10 Minutes

100 Marks

Time: 2:00 – 5:10 pm **Date: 10/06/2015**

Instructions:

1. There are a total of 6 pages in the paper.
2. **There are total 13 questions in the paper. <u>Answer any 10 questions.</u>**
3. You are allowed 10 minutes extra reading time during which you are NOT allowed to write.
4. **<u>Answer each question neatly on a new page in the answer booklet provided and clearly number the question attempted. All relevant working must be shown.</u>**
5. Students may use a calculator, provided it is silent & non-programmable.
6. If you use extra sheets of paper, attach it securely to the answer booklet.
7. Write your student identity number at the top of each sheet used.

Question - 1

a) Solve the IVP using the method of undetermined coefficients :

$$y'' - 4y' - 12y = 3e^{5t}$$ **[6 Marks]**

b) Solve the IVP using separable variable method and bring the answer in implicit form:

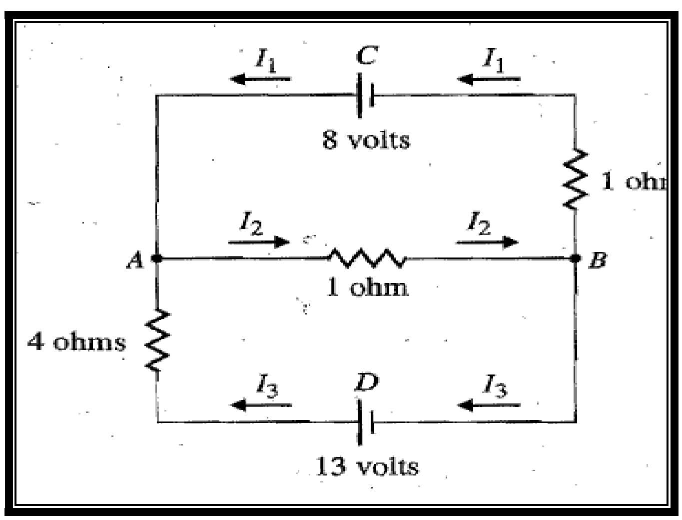
$$y' = \frac{3x^2 + 4x - 4}{2y - 4} \qquad y(1) = 3$$ **[4 Marks]**

Question – 2

A series circuit consists of a resistor with $R = 20\Omega$, an inductor with $L = 1H$, a capacitor with $C = 0.002F$, and a 12 – V battery. If the initial charge and current are both 0, find the charge and current at time t. **[10 Marks]**

Question – 3:

a) Determine the currents I_1, I_2 and I_3 in the electrical network shown in the below figure using Gauss-Jordan elimination method.



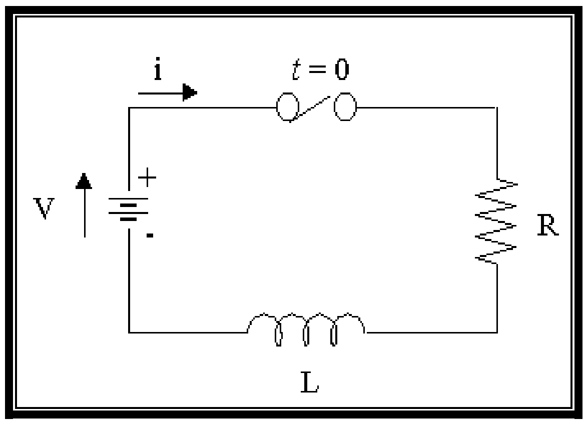
[7 Marks]

b) Express the given vector $w = (1, -8, 12)$ as a linear combination of vectors in

$S = \{(2, -1, 3)\ , (5, 0, 4)\}$ **[3 Marks]**

Question - 4:

a) A series RL circuit with R = 50 Ω and L = 10 H has a constant voltage V = 100 V applied at t =0 by the closing of a switch. Find the equation for current i at t = 0.5 seconds.



[**Hint:** Use the formula $i = \frac{V}{R}(1 - e^{-(R/L)t})$] **[5 Marks]**

b) Solve the first order linear differential equation $\frac{dy}{dx} - \frac{2}{x}y = x$ **[5 Marks]**

Question – 5:

a) Given $f(x, y) = x^3y^5 - 2x^2y + x$, find f_{xxy} and f_{yyy} **[4 Marks]**

b) Use polar coordinates to evaluate $\iint_D 2xy\, dA$; where D is the portion of the region between the circles of radius 2 and radius 5 centered at the origin that lies in the first quadrant.

[**Hint:** $\sin(2\theta) = 2\sin\theta\cos\theta$] **[6 Marks]**

Question – 6:

a) Find the curl and divergence of the vector $F(x,y,z) = yz\,\hat{i} + xz\,\hat{j} + xy\,\hat{k}$ [**4 Marks**]

b) Use Spherical Coordinates to evaluate $\iiint_E \sqrt{x^2+y^2+z^2}\;dV$, where E is bounded below by the cone $\phi = \frac{\pi}{6}$ and above by the sphere $\rho = 2$. [**6 Marks**]

Question – 7:

a) Find the directional derivative of the function $f(x,y) = x^2y^3 - 4y$ at the point $(2,-1)$ in the direction of the vector $v = 2\hat{i} + 5\hat{j}$. [**4 Marks**]

b) Use Green's theorem to evaluate $\oint_C 3xy\,dx + 2xy\,dy$, where C is the rectangle bounded by $x=-2,\ x=4,\ y=1\ \&\ y=2$ and is oriented counter clockwise. [**6 Marks**]

Question – 8:

a) Determine whether $F(x,y) = (\cos y + y\cos x)\,\hat{i} + (\sin x - x\sin y)\,\hat{j}$ is a conservative vector field or not. [**3 Marks**]

b) Use double integral to find the volume of the solid that lies under the plane $3x + 2y + z = 12$ and above the rectangle $R = \{(x,y): 0 \le x \le 1,\ -2 \le y \le 3\}$

[**7 Marks**]

Question – 9:

a) Solve the following Bernoulli's differential equation:

$$\frac{dy}{dx}+\frac{1}{x}\,y=y^3$$ **[6 Marks]**

b) Solve the following Exact differential equation:

$$(2xy-9x^2)\,dx+(2y+x^2+1)\,dy=0$$ **[4 Marks]**

Question – 10:

a) Find $u \cdot v$ and the angle θ formed between the two vectors $u = <1, 2, 0>$ and

$v = <3, -2, 1>$ **[5 Marks]**

b) Let $u = <1,3>, v = <2,1>$ and $w = <4,-1>$. Find the vector x that satisfies

$2u - v + x = 7x + w.$ **[5 Marks]**

Question – 11:

a) If $f(x, y) = x^3 - 4x^2y + y^2$, then find the gradient of f **[3 Marks]**

b) Use Cylindrical coordinates to evaluate the triple integral

$$\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{\sqrt{x^2+y^2}}^{2}(x^2+y^2)\;dz\;dy\;dx$$ **[7 Marks]**

Question – 12:

a) Compute the rank and nullity of the matrix $A = \begin{bmatrix} -2 & -4 & 4 & 5 \\ 3 & 6 & -6 & -4 \\ -2 & -4 & 4 & 9 \end{bmatrix}$ [**7 Marks**]

b) If $a = <4,0,3>$ and $b = <-2,1,5>$. Find $\|a+b\|$ and $2a+5b$ [**3 Marks**]

Question – 13:

Find the Eigen values and the corresponding Eigen vectors of the matrix

$$A = \begin{bmatrix} 2 & 3 & 1 \\ 0 & -1 & 2 \\ 0 & 0 & 3 \end{bmatrix}$$

[**10 Marks**]

THE END