FNU FIJI NATIONAL UNIVERSITY
College of Engineering, Science & Technology

SCHOOL OF ELECTRICAL & ELECTRONICS ENGINEERING

BACHELOR OF ENGINEERING

EEE701 – FIELDS AND WAVES

SEMESTER 1, 2014

DAY/DATE: As timetabled DURATION : Three hours
ROOM: As timetabled

INSTRUCTION TO STUDENTS

1. You are allowed 10 minutes extra reading time during which you are **NOT** to write.
2. Answer ALL questions in Section A and FOUR questions in Section B
3. **Begin the answer to each Question** on a fresh page and use both sides of the sheet.
4. Write clearly the number of the question attempted on the top of each sheet
5. Write your candidate number at the top of each sheet & attach them.
6. Insert all written foolscaps, graph paper etc. in their correct sequence and secure with a string.
7. All sheets of paper on which rough/draft work has been done, cross it through and attach all of them to your answer scripts.
8. Where ever possible, draw clear neat diagrams

Useful vector identities

$$\nabla f = \mathbf{i}\frac{\partial f}{\partial x} + \mathbf{j}\frac{\partial f}{\partial y} + \mathbf{k}\frac{\partial f}{\partial z}$$

$$\nabla \wedge \mathbf{A} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

$$\nabla \wedge (\nabla \wedge \mathbf{A}) = \nabla (\nabla \cdot \mathbf{A}) - \nabla^2 \mathrm{A}.$$

Useful quantities:

$$\varepsilon_0 = 8.854 \text{ x } 10^{-12} \text{ F m}^{-1}$$

$$\mu_0 = 4\,\pi \text{ x } 10^{-7} \text{ H m}^{-1}$$

$$\frac{1}{4\,\pi\,\varepsilon_0} = 9 \times 10^9$$

Section A (Answer ALL questions)

(20 marks)

1. Using rectangular coordinate system, for a scalar field A verify:

$$\nabla \wedge (\nabla A) = 0$$ (4 marks)

2. In a rectangular Cartesian coordinate system, a point charges –8 nC is kept at A (1,2,3)m and another point charge 5 nC are kept at B (3,4,5)m respectively. Find the resultant field at C (1,1,1)m. (5 marks)

3. Defining all the quantities used, write down the vector relationship between electric potential and electric field. (2 marks)

4. Two long parallel wires carry currents of 2 A and 8 A. The distance between the wires is 5 cm. If the currents are in the same direction, where is the point where the magnetic induction field is zero? (3 marks)

5. A long solenoid has n number of turns of coil and a length of ℓ carries a current of I. Using Amperes circuital law Show that the magnetic induction **B** inside the coil is $(\mu_0\, n\, I)/\ell$ (3 marks)

6. Explain the mode of propagation of electromagnetic waves in free space, coaxial transmission lines and twin wire transmission lines. (3 marks)

Section B (Answer FOUR questions only)

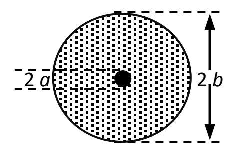
(Each question carries 20 marks)

B1. State Gauss law in electrostatic fields. (2 marks)

i) A charged spherical shell of radius 0.5 m is kept in free space with its centre at the origin. The charge density is 20 nC/m^2. Using Gauss law or otherwise, calculate the electric **E** field at a point **A** (1, 1, 1) m. (4 marks)

ii) a) Using Gauss law or otherwise, derive an expression in cylindrical coordinates for the electric field at an arbitrary point (ρ, ϕ, z) produced by a **long** linear charge distribution ρ_ℓ C m^{-1} kept in air along the Z axis. (4 marks)

b) The diameters of the inner and outer conductors of a long coaxial cable are 2*a* and 2*b* respectively. The outer conductor is earthed and the inner conductor carries a charge of density ρ_ℓ Cm^{-1}. The dielectric constant of the spacer medium between the conductors is ε_r. Derive an expression for the potential difference between the two conductors. (4 marks)



c) Hence derive an expression for the capacitance per unit length of the coaxial cable. (4 marks)

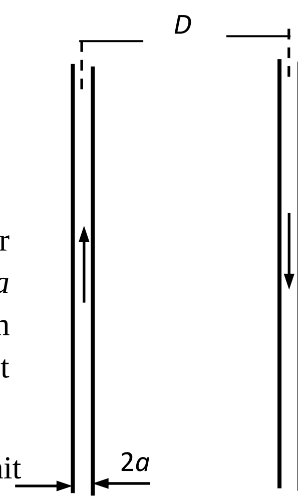
d) The diameters of the inner and outer conductors of a coaxial cable are 0.7 mm and 11.0 mm respectively. The dielectric constant of the spacer medium between the conductors is 2.9. What is the capacitance per unit length of the cable? (2 marks)

B2. State the Amperes circuital law for Magnetic Induction. (3 marks)

a) Using the Amperes circuital law or otherwise, determine the magnetic induction at a point due to a **long** conductor carrying a current I. (5 marks)

b) The Figure shows a section of a twin wire transmission line in air feeding a current to a load. The diameters of the conductors are *2a* and the separation between the centers of the conductors is *D*. Obtain an expression for the Total Magnetic flux linked between a unit length of the conductors. (7 marks)

c) If *2a* = 0.6mm and *D* = 20.5 mm, calculate the inductance per unit length of the twin wire transmission line. (5 marks)



B3. Write down the four Maxwell's equations. (4 marks)

a) Starting with the Maxwell's equations, derive the wave equation for the electric field **E** of electromagnetic (EM) waves in a charge free noncom-ducting dielectric medium of dielectric constant ε_r. (5 marks)

b) If the dielectric constant $\varepsilon_r = 1.05$, calculate the phase velocity of the EM waves. (3 marks)

c) Assume a plane polarized solution $\boldsymbol{E}_x = E_0 \sin(\omega t - k z)\, \boldsymbol{i}$ to the wave equation, show that the electromagnetic waves are propagated through this space in TEM mode. (5 marks)

d) Calculate the characteristic impedance of the medium. (4 marks)

B4. a) In a two conductor transmission line, explain how the electric (**E**) and magnetic (**H**) fields are produced. Considering **one** of the transmission lines, draw a clear diagram showing the **E** and **H** and the direction of propagation of the electromagnetic waves (5 marks)

b) Draw a small section of a transmission line, and mark all the electrical parameters in this section. A high frequency generator $v = v_0\, e^{j\omega t}$ is connected at one end of a transmission line. The current flowing is $i = i_0\, e^{j\omega t}$ Write down the expressions for the change in voltage and current along the line. Hence obtain the "*Telegraphers equation*" for the rate of change of voltage (v) and current (i) along the line. (5 marks)

c) Hence obtain the wave equation for the voltage as:

$$\frac{d^2 v}{d x^2} = (R + j\omega L)(G + j\omega C)v \qquad \text{(5 marks)}$$

d) The propagation constant P for the waves is $P = \sqrt{(R + j\omega L)(G + j\omega C)}$.
For a transmission line L = 250 nH/m and C = 100 pF/m and $R \ll \omega L$ and $G \ll \omega C$. Calculate the phase velocity of the electromagnetic waves on the line. (5 marks)

B5. Figure B5 shows a transmission line of characteristic impedance Z_0 is terminated by a load Z at one end. The origin of the coordinate system is assumed to be at the load.

Figure B5

The voltage and current waves on the transmission line are:

$$v = (A\,e^{-Px} + B\,e^{Px})\,e^{j\,\omega\,t} \text{ and}$$

$$i = \frac{1}{Z_0}(A\,e^{-Px} - B\,e^{Px})\,e^{j\,\omega\,t}$$

i) Show that the voltage reflection coefficient R_v is given by

$$R_v = \frac{Z - Z_0}{Z + Z_0}$$ (5 marks)

ii) Assume $Z_0 = 50\ \Omega$.
Discuss the nature of the reflected wave when

a) $Z = 70\ \Omega$
b) $Z = 30\ \Omega$
c) $Z = 50\ \Omega$ (7 marks)

iii) A complex load of $(30 + j\,60)\ \Omega$ is now connected to the transmission line. Using the Smith chart determine the magnitude and the phase angle of the reflection coefficient. (4 marks)

iv) If the incident voltage wave A is 300 sin ωt, write down the expression for the reflected wave. (4 marks)