



College of Engineering, Science and Technology

School of Electrical and Electronics Engineering

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EEE 410 – Mathematics for Technicians 2

Semester 2

FINAL EXAMINATION

2013

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**Time Allowed: 3 Hours plus 10 minutes reading**

**100 Marks**

**Instructions:**

1. There are total of five questions. All are compulsory.
2. This exam is worth 50% of your overall mark. The minimum exam mark is 50/100.
3. Answer each question neatly on a new page in the answer booklet provided and clearly number the question attempted. All relevant working must be shown.
4. Students may use a calculator, provided it is silent & non-programmable.
5. If you use extra sheets of paper, attach it securely to the answer booklet.

**Question 1 (25 Marks)**

(a) If  $z = 2 - 2i$  and  $w = -1 + 3i$  then evaluate the following:

i.  $|w|$

ii.  $Arg(z)$

iii.  $\bar{w}$

iv.  $z \times w$

v.  $\frac{w}{z}$

(8 Marks)

(b) If  $z = 2 - 2i$  and  $w = -1 + \sqrt{3}i$  then

i. Convert  $z$  and  $w$  into polar form.

ii. Using  $z$  and  $w$  into polar form, find  $\frac{z}{w}$ .

iii. Convert  $z$  into the exponential form of a complex number.

(8 Marks)

(c) Solve  $x^2 - 4x + 13 = 0$ .

(2 Marks)

(d) Use De Moivre's Theorem to evaluate  $(1 - i)^{10}$ .

(3 Marks)

(e) Find the fourth roots of  $-1 - \sqrt{3}i$ .

(4 Marks)

**Question 2 (20 Marks)**

(a) Find the **Laplace transform** of  $f(t) = 2 \sin 3t - \cos 3t$ .

(3 Marks)

(b) Find the **inverse Laplace transform** of  $F(s) = \frac{2}{s} + \frac{4}{s^2} - \frac{72}{s^5}$ .

(5 Marks)

(c) Use the *s-shift theorem* to find  $\mathcal{L}\{e^{-3t} \sin 2t\}$ .

(3 Marks)

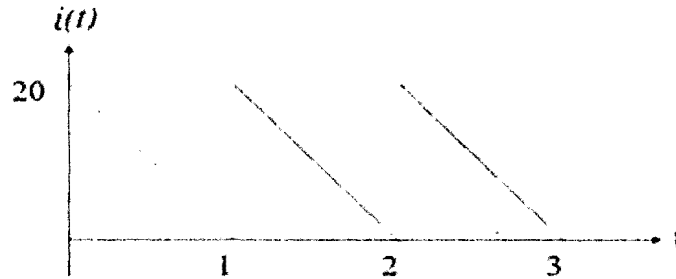
(d) Solve the second order differential equation given below using Laplace transform:

$$y'' + 5y' + 4y = e^{-t} \quad \text{with } y(0) = 1 \text{ and } y'(0) = 0.$$

(9 Marks)

**Question 3 (20 Marks)**

(a) Study the waveform and answer the questions that follow.



- i. What is the general form of the wave between  $t = 0$  and  $t = 1$ .
- ii. Calculate the average value.
- iii. Calculate the RMS value. (6 Marks)

(b) Calculate the voltage across the inductor of inductance 0.2H if the current flowing through the inductor is  $i_L(t) = e^{2t} \cos(10t + \frac{\pi}{6})$ . (5 Marks)

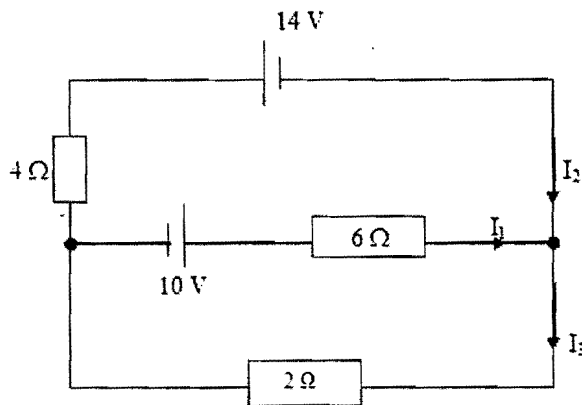
(c) Evaluate the following definite integrals:

- i.  $\int x^2 e^x dx$
- ii.  $\int 2x e^{x^2} dx$  (6 Marks)

(d) If  $y = \frac{x^2}{3x-4}$  then find  $\frac{dy}{dx}$ . (3 Marks)

**Question 4 (20 Marks)**

(a) Find the currents  $I_1$ ,  $I_2$  and  $I_3$  in the multi-loop circuit given using Cramer's rule.



(8 Marks)

(b) In an experiment the following data were recorded:

$\theta$	1	2	3	4
$\beta$	6.75	15.19	34.17	76.89

The relationship is given by:  $\beta = \lambda \kappa^\theta$ .

- i. Determine the type of graph paper to be used (Show all working for it).
- ii. Plot the points and draw the graph.
- iii. From the graph, find the gradient of the line.
- iv. Find the vertical intercept.
- v. What is the relationship between  $\theta$  and  $\beta$ ?

(12 Marks)

**Question 5 (15 Marks)**

The Fourier Series representation of a function  $f$  is

$$f(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos nw_0 t + b_n \sin nw_0 t),$$

where  $w_0$  is the fundamental frequency and  $a_n$  and  $b_n$  are Fourier constants.

Consider the function,

$$f(t) = \begin{cases} 0, & -\pi < t \leq 0 \\ \pi, & 0 < t \leq \pi \end{cases}$$

- (a) Draw the graph for the function for one period.
- (b) Find the fundamental frequency.
- (c) Find the value of  $a_0$ .
- (d) Find the value(s) of the Fourier constants  $a_n$  and  $b_n$ .
- (e) Hence, determine the Fourier Series.

(15 Marks)

**End of Questions**