



College of Engineering, Science and Technology

School of Mathematical & Computing Sciences

EEE 605 – Mathematics for Engineers

Semester 1

FINAL EXAMINATION

2013

Time Allowed: 3 Hours plus 10 minutes reading

100 Marks

Instructions:

1. There are total of nine questions. All are compulsory.
2. This exam is worth 50% of your overall mark. The minimum exam mark is 50/100.
3. Answer each question neatly on a new page in the answer booklet provided and clearly number the question attempted. All relevant working must be shown.
4. Students may use a calculator, provided it is silent & non-programmable.
5. If you use extra sheets of paper, attach it securely to the answer booklet.

Question 1

(a) Find the centre and radius of the sphere $x^2 + y^2 + z^2 - 2x - 4y - 2 = 0$. (2 Marks)

(b) Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ if $z = x^2 \cos(xy^3)$. (4 Marks)

(c) Find the angle between the vectors $\mathbf{u} = \mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$ and $\mathbf{v} = 2\mathbf{i} + 4\mathbf{j} + 3\mathbf{k}$. (4 Marks)

Question 2

(a) Evaluate the double integral

$$\iint_R x^2 y dA$$

over the rectangle $R = \{(x, y): 0 \leq x \leq 1, 2 \leq y \leq 4\}$.

(5 Marks)

(b) Evaluate

$$\iint_R xy dA$$

over the region R enclosed between $y = \sqrt{x}$ and $y = x^2$.

(5 Marks)

Question 3

(a) Use polar coordinates to evaluate $\int_{-1}^1 \int_0^{\sqrt{1-x^2}} (x^2 + y^2) dy dx$. (5 Marks)

(b) Evaluate the line integral

$$\int_C (x + y) ds$$

Over the line $\mathbf{r}(t) = t\mathbf{i} + 2t\mathbf{j}$ where $0 \leq t \leq 1$.

(5 Marks)

Question 4

- (a) Use Green's theorem to evaluate the line integral

$$\oint_C (3x + 6xy - y^3) dx + (4y + 3x^2 - 3xy^2) dy$$

where C is the unit circle oriented counterclockwise.

(5 Marks)

- (b) Use a triple integral to find the volume of the solid within the cylinder $x^2 + y^2 = 9$ and between planes $z = 1$ and $x + z = 5$.

(5 Marks)

Question 5

- (a) Determine whether the set all 2×2 singular matrices with the standard operations is a vector space. If it is not, identify at least one vector axioms that fail.

(5 Marks)

- (b) Determine whether the set S is linearly independent or linearly dependent.

$$S = \{(-2, 4), (1, -2)\}$$

(5 Marks)

Question 6

If A is a 4×5 matrix as given below

$$A = \begin{bmatrix} 1 & 2 & 1 & 0 & 0 \\ 2 & 5 & 1 & 1 & 0 \\ 3 & 7 & 2 & 2 & -2 \\ 4 & 9 & 3 & -1 & 4 \end{bmatrix}$$

Then find:

- (a) rank and nullity of A .
(b) basis for the column space of A .

(10 Marks)

Question 7

Solve the system of linear differential equations.

$$y_1' = 3y_1 + 2y_2$$

$$y_2' = 6y_1 - y_2$$

(20 Marks)

Question 8

Solve the following:

(a) $y' = \frac{x - \sin x}{y^4 + 2}$

(5 Marks)

(b) $(3x^2y)dx + (x^3 + 2y^4)dy = 0$

(5 Marks)

Question 9

(a) Find the divergence and the curl of the vector field

$$F(x, y, z) = xz^3 \mathbf{i} + 2y^4x^2 \mathbf{j} + 5z^2y \mathbf{k}.$$

(7 Marks)

(b) Determine the gradient field of $\Phi(x, y) = 2x + \frac{y^2}{2}$.

(3 Marks)

End of Questions